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December 2020

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Submitted to Review of Scientific Instruments
Interpolating individual line-of-sight neutron spectrometer measurements onto the “sky” at the National Ignition Facility (NIF)

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(Dated: November 17, 2020)

Nuclear diagnostics provide measurements of inertial confinement fusion (ICF) implosions used as metrics of performance for the shot. The interpretation of these measurements for shots with low mode asymmetries requires a way of combining the data to produce a “sky map” where the individual line-of-sight values are used to interpolate to other positions in the sky. These interpolations can provide information regarding the orientation of the low mode asymmetries. We describe the interpolation method, associated uncertainties, and the correlations between different metrics, e.g. Tion, down scatter ratio (DSR) and hot-spot velocity direction. This work is also related to recently reported studies1,2 of low mode asymmetries. We report an analysis that makes use of a newly commissioned line-of-sight, a scheme for incorporating multiple neutron spectrum measurement types, and recent work on the sources of implosion asymmetry to provide a more complete picture of implosion performance.

I. INTRODUCTION

Inertial confinement fusion (ICF) results from the conversion of capsule ablation in a high temperature radiation field into radial compression providing PdV work to increase the temperature of the DT fuel contained within the capsule3,4. The coupling of the PdV work to the fuel compression depends on the symmetry of the implosion with asymmetric components not contributing to the increased temperatures5. Diagnosing implosion performance must include an assessment of low mode asymmetries when comparing to predictions of implosion metrics6.

Exploring the effect of laser drive “up-down” asymmetry on the NIF7,8 in two dimensions lead to a characterization of the capsule response in terms of a spectrum of perturbations described as a Legendre polynomial. The expansion terms of interest to this study have index 0 and 1, and are referred to as “mode 0” or 4π isotropic term, and a “mode 1” term which is anisotropic. This computational study made predictions regarding the angular dependence which are the subject of this study: 1) the burning plasma common velocity, $v_{h.s.}$ indicates the direction of the perturbation, 2) ion temperature $T_{\text{ion}}$ has an angular distribution that varies as $\cos^2 \xi$ defined with respect to the perturbation direction, and 3) the fuel shell areal density, $\rho R$ has an angular distribution that varies as $-\cos \xi$ defined with respect to the perturbation direction (that is, low $\rho R$ in the direction of $v_{h.s.}$, and high $\rho R$ away).

A suite of neutron diagnostics exist to measure these three quantities: burning plasma common velocity (hot-spot velocity), $v_{h.s.}$,9,10, ion temperature $T_{\text{ion}}$, and the down-scattered ratio11–13. These neutron diagnostics are positioned at NIF along independent line-of-sight (LOS) directions shown in

![Fig. 1. Neutron spectrometer system at NIF. The 7 line-of-sight measurements, 6 using neutron time-of-flight, 1 using neutron energy, are arranged around the Target Chamber system. The time-of-flight systems are roughly 20 meters from the Target Chamber Center (TCC). The system of spectrometers can be used to determine 3 dimensional features of imploding ICF capsules.](image)

Fig. 1 and the work reported here combines these measurements into a three dimensional reconstruction describing the interpolation of the measured quantities to all angles, and infers directional information related to mode 1 perturbations, testing the calculations.
II. OBSERVABLES

Each line-of-sight neutron spectrometer reports the three quantities of interest: $v_{\text{LOS}}$, $T_{\text{ion}}$, and DSR (down-scattered ratio). These spectroscopic observables are connected to the underlying plasma variables: “thermal” ion temperature, ion kinetic energy, and plasma velocity which are an average over the plasma in space and time weighted by the ion reactivity, the so called “burn-weighted average” measured by the spectrometers\textsuperscript{14} (denoted by the brackets $\langle \cdot \rangle$). The analysis measures the shift of the mean neutron kinetic energy:

$$\langle \omega_{\text{LOS}} \rangle = \langle u_{i0} \rangle + \langle \kappa \rangle + \frac{2 + \beta_{0}^{2}}{2v_{0}} \langle \tau \rangle + \cdots \quad (1)$$

where $u_{i0}$ is the center-of-momentum motion of the fusing ion pair in the direction of of the LOS, $\kappa$ is the shift due to the burn averaged kinetic energy distribution of the reacting ions, $\tau$ is the ion thermal temperature in units of velocity squared. The neutron velocity $v_{0}$ is taken to be $51,233.592(34)$ km/s for DT fusion and $21,601.8596(77)$ km/s for DD fusion (using the CODATA 2010 values\textsuperscript{15}). The relativistic velocity $\beta_{0} = v_{0}/c$ where $c$ is the speed-of-light. Higher order terms are neglected in this analysis.

The $T_{\text{ion}}$ is associated with the variance of the neutron kinetic energy distribution:

$$\text{Var}(\omega_{\text{LOS}}) = \langle \tau \rangle + \text{Var}(u_{i0}) + \cdots \quad (2)$$

which shows the source of departure of the neutron variance from the burn averaged ion thermal temperature where the plasma velocity variance is large\textsuperscript{16}.

The DSR, down scatter ratio is the integral of the neutron spectrum between 10 and 12 MeV to 13 to 15 MeV. The neutrons produced by fusion reactions in the plasma will be transmit through a high areal density region composed of the DT fuel layer and the remaining C, CH or Be capsule. These neutrons can undergo elastic scattering through angles that put them along the detector LOS, but with reduced energy:

$$\cos \theta_{\text{CM}} \approx 1 - \frac{1}{2} \left( 1 - \frac{K_{n}'}{K_{n}} \right) \frac{M}{m_{n}} \left( 1 + \frac{m_{n}}{M} \right)^{2} \quad (3)$$

where there is a term for each scattering ion. For this analysis, we consider scattering from T and D only, where there are equal number of atoms for the two ion species. There are additional complications due to the distribution of the neutron emission distribution, and the areal density of the fuel shell due to thickness variation. Combining these various distributions will blur out the scattering annulus into a disk for neutrons originating from fusions distributed through out the hot-spot. The resulting region is shown in Fig. 2b. For each measurement these disks have a 60° angular extent from the LOS direction and can overlap with disks associated with other LOS directions. The overlapping regions correlate one measurement to another and fitting the DSR measurements require properly accounting for the correlations. Table I shows the correlations found in layered DT implosions on the NIF and the calculations assuming a uniform scattering layer and either a distributed or point source model.

III. INTERPOLATION

Angular variations in the plasma velocity variance and DSR aligned with the hot-spot velocity are predicted\textsuperscript{7} for mode 1 capsule drive perturbations. These variations will be measured along each of the LOS and these measurements can be used, along with a model of the angular variations for each observable. The hot-spot velocity Eq. 1 suggests the LOS ve-
The dot product expands to:

\[ \sum_{\text{dist.} \times \text{point}} \]

where the axis of symmetry is defined by the unit vector:

\[ \hat{n}_{\text{LOS}} = \hat{v}_{\text{LOS}} \cdot \hat{a}_{\text{LOS}} \]

with the first term on the r.h.s. corresponding to the first term in Eq. with \( \hat{v}_{\text{LOS}} \) the hot-spot velocity vector and \( \hat{a}_{\text{LOS}} \), the LOS unit direction vector. The second term, \( \hat{v}_{\text{LOS}} \), the isotropic velocity, corresponds to the second and third terms of Eq. 1 associated with the temperature dependent shift of the mean neutron kinetic energy distribution. The \( T_{\text{iso}} \) distribution follows Eq. 2:

\[ T_{\text{iso}}(\theta, \phi) = T_0 + \Delta T \cos^2 \xi(\theta_0, \phi_0, \theta, \phi) \]

where the angle \( \xi \) is defined by the \( T_{\text{iso}} \) axis of azimuthal symmetry (an assumption of this model) and some sky location (e.g. the \( i \)-th LOS directions):

\[ \cos \xi(\theta_0, \phi_0, \theta, \phi) = \hat{n}_{\text{LOS}} \cdot \hat{a}_{\text{LOS}} \]

where the axis of symmetry is defined by the unit vector:

\[ \hat{n}_{\text{LOS}} = (\sin \theta_0 \cos \phi_0, \sin \theta_0 \sin \phi_0, \cos \theta_0) \]

where \( \hat{a}_{\text{LOS}} \) and \( \hat{n}_{\text{LOS}} \) are found by optimizing the \( \chi^2 \) formed by the interpolation: \( \chi^2 = \sum_{i} \frac{1}{\sigma^2_i} [T_i - T_{\text{iso}} - \Delta T \cos^2 \xi(\theta_0, \phi_0, \theta, \phi)]^2 \)

The DSR angular dependence is described by:

\[ \text{DSR}(\theta, \phi) = \sum_{L=0}^{L=2} \sum_{M=-L}^{L} a_{LM} Y_{LM}(\theta, \phi) \]

a truncated spherical harmonic expansion. The four parameters: \( a_{0,0}, a_{1,-1}, a_{1,0}, a_{1,1} \), define the DSR sky.

### IV. OPTIMIZATION

Given the LOS measurements, the parameters to the three quantities: hot-spot velocity \( v_{\text{LOS}} \), \( T_{\text{iso}} \), and DSR in Eqs. 5, 6 and 10 are chosen to be described by four parameters because of the limited number of LOS measurements available and the desire to describe an effective “direction” for the variation. The DSR variation at \( L = 0, 1 \) can be described as the direction defined by the maximum and minimum DSR values interpolated in the DSR sky.

### V. RESULTS

The analysis is applied to the NIF shot N190918-001, a cryo-layered shot of the “HyE” ICF campaign. Fig. 3 shows the result of the hot-spot velocity measurements fit to the LOS velocities. In this fit 4 parameters are determined by 4 measurements. Additional measurements (primarily from NITOF...
and SPEC-PL) will provide a means of using the $\chi^2$ value as a test of the hot-spot velocity model.

The $T_{\text{in}}$ sky is shown in Fig. 4 and shows the expected large velocity variance roughly aligned with the hot spot velocity.

The DSR sky shown in Fig. 5 captures the three determinations of mode 1 perturbation discussed above. The general signatures being a hot-spot velocity the direction of which indicates an axis of symmetry for both the DSR distribution (which is “anti-aligned”) and the $T_{\text{in}}$ distribution which peaks along the axis. The net capsule drive asymmetry derived from the laser, diagnostics windows, ice layer and ablator is shown as the red triangle symbol\(^{18}\). The correspondence of these independent analyses provides evidence that supports the mode 1 perturbation model.

Table II shows the directions from each of the four independent analyses. Combining the directions from the 3 observable shows agreement with the average directions $\delta \theta = 13^\circ$ and $\delta \phi = 18^\circ$. When combining these directions with the calculated perturbation direction the agreement is $\delta \theta = 12^\circ$ and $\delta \phi = 16^\circ$. The large mode 1 perturbation for shot N190918-001 is attributed to a large ablator capsule mode 1, which was determined (after the shot) by the inspection data for the specific capsule.

VI. DISCUSSION

The RTNAD diagnostics at NIF\(^{19}\) provides a check of the DSR sky map shown in Fig. 5. The RTNADs measure the neutron activation along 40 LOS directions and show a decrease

### TABLE II. Summary of the axis of symmetry directions for the four independent analyses. The uncertainties are taken from the 1σ variation in the optimization. The combined averages use the r.m.s. as an indication of the variation of the results.

<table>
<thead>
<tr>
<th>analysis</th>
<th>$\theta$</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>hot-spot velocity</td>
<td>$125^\circ \pm 2^\circ$</td>
<td>$235^\circ \pm 3^\circ$</td>
</tr>
<tr>
<td>$T_{\text{in}}$</td>
<td>$157^\circ \pm 46^\circ$</td>
<td>$192^\circ \pm 11^\circ$</td>
</tr>
<tr>
<td>DSR</td>
<td>$135^\circ \pm 47^\circ$</td>
<td>$219^\circ \pm 103^\circ$</td>
</tr>
<tr>
<td>average</td>
<td>$139^\circ \pm 13^\circ$</td>
<td>$215^\circ \pm 18^\circ$</td>
</tr>
<tr>
<td>drive asymmetries</td>
<td>$149^\circ \pm 9^\circ$</td>
<td>$223^\circ \pm 12^\circ$</td>
</tr>
<tr>
<td>average all</td>
<td>$142^\circ \pm 12^\circ$</td>
<td>$217^\circ \pm 16^\circ$</td>
</tr>
</tbody>
</table>
activation in regions of high DSR. The RTNAD sky map fit to a spherical harmonic expansion to $L=2$ is shown in Fig. 6. The regions of high and low activation correspond to the DSR map from Fig. 5 low and high regions. This correspondence suggests the possibility of including more diagnostics into a simultaneous fit of the mode 1 perturbation analysis, such as the RTNAD. The model for such an analysis would constraint the various diagnostic reference system axes to be the same. The inclusion of the RTNAD data, as well as the drive asymmetry data coupled with the model for drive asymmetry could be used to increase the precision of the signatures of mode 1 perturbations and provide insight into capsule implosion performance. Building a “mode 1 model,” including the relationship among the parameters of Eqs. 5-10 would provide a classification of shots, those consistent with a mode 1 model and those inconsistent with the model, which provides valuable diagnostic data for assessing shot performance.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

ACKNOWLEDGMENTS

Lawrence Livermore National Laboratory is operated by Lawrence Livermore National Security, LLC, for the U.S. Department of Energy, National Nuclear Security Administration under Contract No. DE-AC52-07NA27344. This article (LLNL-PROC-817440) was prepared as an account of work sponsored by an agency of the U.S. government. Neither the U.S. government nor Lawrence Livermore National Security, LLC, nor any of their employees make any warranty, expressed or implied, or assume any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represent that its use would not infringe privately owned rights. The views and opinions of authors expressed herein do not necessarily state or reflect those of the U.S. government or Lawrence Livermore National Security, LLC, and shall not be used for advertising or product endorsement purposes.