Dimensionless Parameter Scaling of Intrinsic Torque in C-Mod Enhanced Confinement Plasmas

J.E. Rice$^1$, N.M. Cao$^1$, T. Tala$^2$, C. Chrystal$^3$, M.J. Greenwald$^1$, J.W. Hughes$^1$, E.S. Marmar$^1$, M.L. Reinke$^4$, P. Rodriguez Fernandez$^1$ and A. Salmi$^2$

$^1$ PSFC, MIT, Cambridge, Massachusetts 02139, USA
$^2$ VTT, Box 1000, FIN-02044, Finland
$^3$ General Atomics, San Diego, California, USA
$^4$ ORNL, Oak Ridge, Tennessee 37831, USA

September 2020

Plasma Science and Fusion Center
Massachusetts Institute of Technology
Cambridge MA 02139 USA

This work supported at MIT by DoE Award DE-SC0014264. This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Fusion Energy Sciences, using the DIII-D National Fusion Facility, a DOE Office of Science user facility, under Award DE-FC02-04ER54698. This work has been partially carried out within the framework of the EUROfusion Consortium and has received funding from the Euratom research and training programme 2014–2018 and 2019–2020 under Grant Agreement No. 633053. Reproduction, translation, publication, use and disposal, in whole or in part, by or for the United States government is permitted.

Submitted to Nuclear Fusion
Dimensionless Parameter Scaling of Intrinsic Torque in C-Mod Enhanced Confinement Plasmas

J.E. Rice, N.M. Cao, T. Talala, C. Chrystal, M.J. Greenwald, J.W. Hughes, E.S. Marmar, M.L. Reinke, P. Rodriguez Fernandez and A. Salmi

PSFC, MIT, Cambridge, Massachusetts 02139, USA

♯ VTT, Box 1000, FIN-02044, Finland

♭ General Atomics, San Diego, California, USA

† ORNL, Oak Ridge, Tennessee 37831, USA

Abstract

A dimensionless parameter dependence study of intrinsic torque has been performed on a database of H- and I-mode plasmas from the Alcator C-Mod tokamak. The torque was determined by comparing intrinsic angular momentum density profiles just before and just after L-H and L-I transitions. The intrinsic torque has been found to scale as $\beta N^{1.5} \rho_s^{-1.0} \nu_s^{0.1}$, with the parameter ranges $0.3 \leq \beta N \leq 1.5$, $0.004 \leq \rho_s \leq 0.011$ and $0.04 \leq \nu_s \leq 0.9$. Comparison with results from JET and DIII-D suggests that the intrinsic torque should be normalized by some measure of the device size. Depending upon this normalization, the estimated total intrinsic torques for ITER, SPARC and ARC are $\sim 20$, $\sim 4$ and $\sim 8$ Nm, respectively.
1. Introduction

The benefits of plasma rotation and its gradient to tokamak performance have been well established. Rotation can suppress resistive wall modes [1, 2] and velocity shear can break up turbulent eddies, leading to edge [3, 4, 5] or internal [6, 7] transport barriers. The rotation velocity spatial profile is a result of a complicated combination of many competing effects [8, 9], including self generated flow, known as intrinsic rotation. Since modern gyro-kinetic codes are ill equipped to capture the intrinsic velocity profile (including the sign) from first principles and there yet exists no comprehensive theory of self generated flow with sound predictive capability, extrapolation to future devices has relied upon scaling studies across several tokamaks. Early results have indicated the association of the co-current intrinsic rotation velocity in enhanced confinement regimes with a measure of plasma performance, such as the global stored energy, with an inverse dependence on plasma current [10]. An investigation of C-Mod plasmas has identified the pedestal temperature gradient [11] as the drive for intrinsic rotation and these results have been compared favorably [12] to a model of fluctuation entropy balance in an ITG turbulence dominant regime [13]. The observed scaling of the intrinsic rotation \( \propto \nabla T^{0.8} \varrho_{95}^{1.0} / B^{0.7} \) [12] agrees quantitatively with the calculated Mach number \( M_i \approx \frac{1}{2} \rho_s L_s / L_T \) [13], where \( \rho_s \) is the normalized ion gyro-radius and \( L_s \) and \( L_T \) are the magnetic shear and temperature gradient scale lengths, respectively. This model dependence has yet to be compared to observations from other devices. The association of the temperature gradient with intrinsic torque was originally pointed out in JFT-2M experiments [14]. The origin of the intrinsic torque lies in the residual stress through its dependence on the temperature gradient [15, 16].

A different approach to the study of intrinsic rotation involves modulating torque balanced neutral beam injection [17, 18, 19, 20, 21]. A dimensionless scaling of the intrinsic torque, \( T_{int} \), determined by integrating over the profiles in DIII-D plasmas using the onion peeling method [19] found that the intrinsic torque normalized to the ion temperature is proportional to \( \rho_s^{-1.5} \nu_s^0.26 \pm 0.04 \) [22]. This same methodology has been applied to multiple devices [21, 23]. In discharges with matched collisionality \( \nu_s \) and normalized pressure \( \beta_N \)(also \( q \) and \( T_e/T_i \)), this investigation, using standard dimensionless parameter scan methods [24, 25], also yields an inverse relation of \( T_{int} \) to the normalized gyro-radius. The precise dependence of \( T_{int} \) on the normalized gyro-radius is of interest in the extrapolation to future devices with unexplored values of \( \rho_s \). One open area of understanding concerns the normalization of the intrinsic torque when using this dimensionless variable approach.

In order to explore different dimensional parameter ranges and to shed light on this normalization issue, in this paper the intrinsic torque in C-Mod plasmas will be added to the multiple device study. The determination of \( T_{int} \) in the C-Mod case relies upon a very direct method: comparison of the intrinsic angular momentum density profiles before and after H- and I-mode transitions from L-mode. The evolutionary time scale determines the total intrinsic torque, without the use of external momentum input. The new contributions from C-Mod will be summarized as follows. Section 2 will cover the experimental setup and will describe the measurement of \( T_{int} \). In section 3 will be shown the \( \nu_s \), \( \beta_N \) and \( \rho_s \) scaling results arising from a database of \( \sim 300 \) individual H- and I-mode discharges. A comparison with findings from other devices, followed by a
discussion and conclusions, will be presented in section 4.

2. Experimental Setup and Determination of Intrinsic Torque

The present results were obtained from the Alcator C-Mod tokamak [26, 27], a compact (R = 0.67 m), high magnetic field (B_T \leq 8.1 T) device. ICRF heating power up to 5 MW [28] allows access to H-mode [29, 27] and I-mode [30, 31], both enhanced confinement regimes which feature a steep edge temperature gradient. By changing the plasma current (between 0.46 and 1.7 MA), gas puffing and ICRF power (1-5 MW) input, and varying the toroidal magnetic field (2.7 - 7.8 T), a range of core electron (1.1 - 6.3 keV) and ion (1.1 - 3.8 keV) temperatures has been explored, with line average n_e between 0.8 and 4.8 \times 10^{20}/m^3. This leads to intervals of core dimensionless parameters with these ranges: 3.0 < q_{95} < 6.7, 0.3 < \beta_N < 1.5, 4 < 1000 \times \rho_e < 11 and 0.04 < \nu_e < 0.9. Electron density and temperature profiles were measured using Thomson scattering and electron cyclotron emission [32, 33]. \n
Z_{\text{eff}}\text{, which enters into the collisionality, was determined from visible bremsstrahlung} [33]. Impurity toroidal rotation velocity and ion temperature profiles were determined from an imaging x-ray spectrometer system [34, 35] viewing H- and He-like argon emission. Sample plasma parameter evolution for a representative 5.8 T, 1.1 MA (q_{95} = 3.7) I-mode discharge is shown in Fig.1. Following application of 3 MW of ICRF power, the transition from L- to I-mode featured substantial increases in the electron and ion temperatures and a co-current increment of the toroidal rotation velocity. The core velocity increase has been traced to the rapid formation of the strong pedestal temperature gradient [11].

Full radial profiles for the steady L- and I-mode phases of this discharge are shown in Fig.2. While there is very little change in the density profiles in either regime (a beneficial characteristic of I-mode), there are significant increases in the temperatures and velocity (co-current) across the entire plasma following the transition from L- to I-mode. Notice that the interior velocity profiles are quite flat. Using density and velocity profiles, the angular momentum may be determined from the moment of inertia of a torus as L_o \sim 2\pi^2 \kappa m_D R^2 a^2 \nu with elongation \kappa (which did not change in time during any of the discharges presented here), major radius R and m_D the deuterium mass. Integrating the L_o profiles leads to the total angular momentum \( L_o^{tot} \) shown in the 5th frame of Fig.1. Dividing by the time for the profiles to evolve leads to the total intrinsic torque as \( T_{int} = \Delta L_o^{tot}/\Delta t \). In this case from the initial slope of the total angular momentum, a change in \( L_o^{tot} \) from \(-0.005\) to \(0.015\) Nms in 0.08 s yields a total intrinsic torque of \(0.25\) Nm. Utilizing the initial slope in \( L_o^{tot} \) following the L-I (or L-H) transition avoids having to apply the correction [19]

\[
\frac{dL_o}{dt} = T_{int} - \frac{L_o}{\tau_o}
\]

where \(\tau_o\) is the momentum confinement time. For C-Mod enhanced confinement plasmas, \(\tau_o\) is in the range from 75 to 150 ms [36]. During the initial rise phase \( L_o^{tot} \) is small, ranging from \(-0.005\) Nms at 0.63 s to 0 Nms at 0.65 s to \(+0.004\) Nms at 0.66
Figure 1: From top to bottom, time evolution of average electron density, central electron and ion temperatures (with ICRF power in MW), normalized pressure $\beta_N$, normalized ion gyro-radius $\rho_\ast$ ($\times 1000$), total angular momentum and central toroidal velocity for a 1.1 MA, 5.8 T I-mode plasma. The L- to I-mode transition occurred at 0.61 s.
Figure 2: From top to bottom, spatial profiles of the electron density, electron (from Thomson scattering) and ion temperature, normalized ion gyro-radius ($\times 1000$) and toroidal velocity are shown before (green dashed lines) and well after (solid red lines) the I-mode transition for the discharge of Fig.1. (The ion temperature profiles are lower than those of the electrons, and do not extend to the edge.)
Taking $\tau_\phi = 0.1$ s, this correction varies from $-0.05$ Nm to $0$ Nm to $+0.04$ Nm. Given that this correction is relatively small, and that the sign can vary, the caveat of neglecting this term will be recognized as contributing a 20% uncertainty in the intrinsic torque values quoted here. Another advantage in avoiding this correction is that the scaling of $\tau_\phi$ with plasma parameters is unknown. This method for determination of the intrinsic torque has been applied to the same H- and I-mode database of $\sim 300$ discharges as was previously evaluated for scaling of the intrinsic Mach number [12].

It should be noted that there is a flip in the intrinsic torque in going from L-mode to the ITB phase in stellarator plasmas [37]. There is also a change in the intrinsic torque in C-Mod ITB plasmas formed with off-axis ICRF heating in H-mode plasmas [38], which has been associated with the peaking of the density profile. This occurs on a much longer time scale than seen in Fig.2 and as such this torque is more than a factor of 5 lower than the cases studied here.

3. Observed Intrinsic Torque Scalings

The total intrinsic torque is found to vary from 0.04 to 0.6 Nm for the range of operational parameters. Shown in Fig.3 is the intrinsic torque as a function of line average collisionality for a large body of H- (green) and I-mode (red) discharges. While the I-mode points occupy the low collisionality region of the plot, the range of intrinsic torque is the same for both regimes. Organization with collisionality is not apparent, as shown by the two curves. A similar weak dependence on collisionality was found for DIII-D plasmas [22].

There is a much stronger correlation with volume averaged $\beta_N$ as shown in Fig.4 for those discharges with core $\rho_*$ in the narrow range between 5.5 and $6.5 \times 10^{-3}$. I-mode and H-mode points are interspersed although H-mode plasmas extend to higher $\beta_N$ values. The best power law fit, $\beta_N^{1.4}$, is shown by the dashed line, which is very similar to the best linear fit shown by the solid line. Interestingly, this dependence on $\beta_N$ is identical to the scaling of Mach number found from a multi-device comparison [10]. A possibly important point is that the $\beta_N$ values are for the H- and I-mode phases of the discharges, and it should be noted that the values in L-mode can be substantial, as seen in the $3^{rd}$ panel of Fig.1. It should be noted that intrinsic rotation can be affected by MHD activity through neo-classical toroidal viscosity and electro-magnetic torques [8, 9]. All of the cases described here had sawtooth oscillations, but otherwise no measurable MHD activity.

A range in $1/\rho_*$ between 90 and 250 has been explored by changing the magnetic field and ICRF power. Shown in Fig.5 is $T_{int}$ as a function of core $\rho_*$ for various ranges in volume averaged $\beta_N$. Here $\rho_*$ is evaluated in the plasma core, and as can be seen in Fig.2, there is only a weak dependence on radius. In all cases there is a significant drop as $\rho_*$ is increased, best described by $\rho_*^{-1.0}$. This is similar to the scaling found in DIII-D plasmas [22], although in that case it was the intrinsic torque normalized to the ion temperature.

Combining the results shown in Figs.3, 4 and 5, the best power law for the intrinsic torque in the C-Mod database is $T_{int} \propto \beta_N^{1.5} \rho_*^{-1.0} \nu_*^{0.1}$, and this is compared to
Figure 3: Total intrinsic torque as a function of average collisionality $<\nu_*>$ for I- (red) and H-mode (green) discharges. The solid line is the best linear fit while the dashed line is proportional to $\nu_*^{-0.09}$, the best power law fit.
Figure 4: The observed intrinsic torque as a function of volume averaged $\beta_N$ with $\rho_*$ between $5.5 \times 10^{-3}$ and $6.5$ for H- (green) and I-mode (red) plasmas. The best linear fit (solid line) and power law fit ($\propto \beta_N^{1.4}$) are shown for comparison.
Figure 5: Top frame: the intrinsic torque as a function of $\rho_*$ for ranges of $\beta_N$ between 1.0 and 1.2 (purple dots), and between 0.6 and 0.8 (mustard asterisks), with best power law fits (purple dashed and dotted mustard lines). Bottom frame: ranges of $\beta_N$ between 1.2 and 1.4 (red squares and dotted line), 0.8 and 1.0 (black dots and solid line) and 0.4 and 0.6 (green diamonds and dashed line). For each group of volume averaged $\beta_N$, the data points have been binned and averaged.
the observations in Fig.6. There is slightly more scatter here compared to the intrinsic

given to the intrinsic torque, for example the pedestal temperature gradient. Another source of the scatter could be due to hidden variables not considered, which could have an influence on plasma rotation, such as wall conditioning, plasma shape or ICRF coupling.

4. Discussion and Conclusions

A multi-machine study [23, 21] found that with matched $\nu_s$ and $\beta_N$, the intrinsic torque decreases inversely with $\rho_s$. An open question regards the appropriate normalization of the intrinsic torque. Comparison of the C-Mod results to the $\rho_s$ scan of JET (R = 2.96 m) and DIII-D (R = 1.66 m) [23, 22, 21] is shown in Fig.7. The JET and DIII-D points had matched shape (for DIII-D an elongation of 1.69, upper and lower
Figure 7: The intrinsic torque as a function of $\rho_*$ for JET (red squares), DIII-D (purple asterisks) and C-Mod (green dots) for plasmas with $\beta_N$ near 1.3. The curve is proportional to $\rho_*^{−1.5}$. 
triangularity of 0.14 and 0.34, respectively) with \(q_{95} \sim 3.5\), \(\beta_N \sim 1.3\) and \(T_e/T_i \sim 1\). The C-Mod discharges were not from a matched set, but featured \(q_{95} \sim 3.3\), \(T_e/T_i \sim 1.2\), elongation of 1.7 and upper and lower triangularity in the ranges of 0.2 – 0.4 and 0.52 – 0.77, respectively, with \(\beta_N\) between 1.2 and 1.4. While an eyeball fit \(\rho^* - 1.5\) scaling does a reasonable job capturing the results from the two larger devices, the C-Mod points are about a factor of 5 lower. This difference may be due to the different methodology utilized (including ignoring the \(\tau_\phi\) correction term in Eq.1), the fact that the C-Mod points were taken at a higher collisionality, that the JET and DIII-D \(\rho_*\) values were evaluated at the pedestal top while for C-Mod they were from the plasma core (using the pedestal \(\rho_*\) for C-Mod would shift the points to the left), or that the C-Mod \(\beta_N\) values were of the final state rather than the target L-mode plasma. Other candidates are that \(q\) and \(T_e/T_i\) values were not exactly matched, that the edge plasma is evolving immediately following the transition out of L-mode or that impurity and ion rotation has not yet equilibrated. Given that the C-Mod I- and H-mode results are similar, the influence of a particle pinch, which is quite different between I- and H-mode, is probably not an important factor when evaluating the torque. Another issue is that the JET and DIII-D points were from plasmas with matched shape, while C-Mod was not exactly matched. The effect of plasma shaping and the temperature ratio on the intrinsic torque is unknown and may contribute to the scatter. A final possibility is because of a lack of proper normalization of the intrinsic torque, and the following discussion will explore that hypothesis. Normalization to the ion temperature has been proposed \cite{23, 22, 21} and this multi-machine comparison is presented in Fig.8. Compared to Fig.7, the C-Mod points are closer to the main group, but still about a factor of 3 too low. This suggests that a different normalization should be considered.

A normalization which includes the device size will now be explored. These are not proper dimensionless torques, but will serve to demonstrate possible size scalings. If a normalization proportional to major radius \(R\) is used (the ratio of the angular frequencies), the device comparison appears as in Fig.9. In this case there is much better overlap with the C-Mod points. This curve allows for estimates to be made for future devices (with \(\beta_N \sim 1.3\)). For ITER, with \(R = 6.2\ m\) and \(\rho_* = 0.0015\), the intrinsic torque is estimated to be \(\sim 20\ Nm\), which is somewhat close to the value of 33 Nm predicted in \cite{22} using a normalization proportional to the ion temperature. For SPARC \cite{39}, with \(R = 1.85\ m\) and an estimated \(\rho_* = 0.0027\), Fig.9 gives an intrinsic torque of \(\sim 4\ Nm\), while for ARC \cite{40}, with \(R = 3.3\ m\) and \(\rho_* = 0.002\), \(T_{int}\) is expected to be \(\sim 8\ Nm\).

With a normalization quadratic in \(R\) (since the moment of inertia is proportional to \(R^2\)), the scaling with \(\rho_*\) disappears, and the magnitude of the torque is the same for all devices, as shown in Fig.10. The \(T_{int}/R^2\) values are all around 0.7 N/m, independent of \(\rho_*\). The predictions for ITER and ARC are still around 27 and 8 Nm, respectively, while the SPARC value of 2.5 Nm is somewhat lower.

In summary, investigations of velocity profile changes following L-H and L-I transitions in C-Mod have yielded an expression for the intrinsic torque \(T_{int} = 2.3 \beta_N^{1.5} \rho_*^{-1.0} \nu_*^{0.1} \) Nm. What physics conclusions can be drawn from this? \(\nu_*\) is a measure of the importance of trapped electron modes compared to the effects of tight electron-ion coupling. A weak dependence on \(\nu_*\) implies that TEMs are not particularly important for driving co-current intrinsic rotation, at least for these regimes with steep edge
Figure 8: The intrinsic torque normalized to the ion temperature as a function of $\rho_*$ for JET (red squares), DIII-D (purple asterisks) and C-Mod (green dots) for plasmas with $\beta_N$ near 1.28. The curve is proportional to $\rho_*^{-1.0}$.
Figure 9: Same legend as in Fig.8, except the intrinsic torque is normalized to the device major radius. The curve is proportional to $1/\rho_\star$. 
Figure 10: Same legend as in Fig.8, except the intrinsic torque is normalized to $R^2$. 
temperature gradients. The strongest dependence is on \( \beta_N \), which can be regarded as a response to plasma performance or confinement, at least below certain MHD limits. \( 1/\rho_\ast \) is the number of ion gyro-radii that fit inside the device. For small values (large \( \rho_\ast \)) one would expect that boundary phenomena (including MHD activity and the edge pedestal) to dominate over the effects described in the gyrokinetic model. Interpretation of the significance of \( \rho_\ast \) in driving intrinsic rotation is apparently complicated by the seemingly contradictory results that the intrinsic rotation Mach number was found to scale proportionally with \( \rho_\ast \) [12] while the intrinsic torque depends inversely with \( \rho_\ast \) [22]. Reconciliation can be found through comparison of dimensional [12] and dimensionless analysis of C-Mod data. The change in velocity across H- and I-mode transitions was found to scale as \( \nabla T^{0.8} q_{95}^{1.0} / B^{0.7} \) (here the pedestal temperature gradient). The intrinsic torque is computed from the velocity change as

\[
T_{\text{int}} \sim \frac{\Delta n v}{\Delta t} \sim \frac{n \Delta v}{\Delta t} \sim \frac{n \nabla T^{0.8} q_{95}^{1.0}}{\Delta t B^{0.7}} \sim \frac{n \nabla T^{0.8} B^{0.3}}{\Delta t I^{0.5}}.
\]  

(2)

where \( \Delta t \) is the time for the velocity profile to change. The strongest dependences are on \( n \), \( \nabla T \) and the plasma current. Compare the results of Section 3:

\[
T_{\text{int}} \sim \frac{\beta_N^{1.5}}{\rho_\ast} \sim \frac{\beta_I^{1.5} B^{1.5} B}{T^{1.5} \sqrt{T}} \sim \frac{n^{1.5} T^{1.5} B^{2.5}}{B^{3} \sqrt{T} I^{1.5}} \sim \frac{n^{1.5} T B^{0.5} I^{1.5}}{B^{0.5} I^{1.5}} \sim \frac{n^{1.5} \nabla T}{B^{0.5} I^{1.5}}.
\]  

(3)

The strongest relationship is similarly on \( n \), \( \nabla T \) and the plasma current, with differences attributed to the fact that parameter dependences of \( \Delta t \) are unknown. The bottom line is that \( \rho_\ast \) may not be the best dimensionless ordering parameter.

In the absence of a strong theoretical or gyrokinetic code based underpinning for the understanding of the origins of the intrinsic torque, extrapolation to future devices relies upon multi machine parameter scans [21]. To make use of these in dimensionless parameter studies, it is crucial to determine the proper normalization for the intrinsic torque [21]. The present investigation suggests that the device size is important for this.

5. Acknowledgements

The authors appreciate enlightening discussions with D. Whyte and J. Citrin, and thank J. Irby, Y. Lin, S. Wukitch and the Alcator C-Mod operations and ICRF groups for expert running of the tokamak. Work supported at MIT by DoE Award DE-SC0014264. This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Fusion Energy Sciences, using the DIII-D National Fusion Facility, a DOE Office of Science user facility, under Award DE-FC02-04ER54698. This work has been partially carried out within the framework of the EUROfusion Consortium and has received funding from the Euratom research and training programme 2014-2018 and 2019-2020 under grant agreement No 633053. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

Disclaimer: This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor
any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

References