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D. Brunner, A.Q. Kuang, B. LaBombard, and J.L. Terry

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Plasma Science and Fusion Center
Massachusetts Institute of Technology
Cambridge MA 02139 USA

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The dependence of divertor power sharing on magnetic flux balance in near double-null configurations on Alcator C-Mod

D. Brunner*, A.Q. Kuang, B. LaBombard, and J.L. Terry
Plasma Science and Fusion Center, Massachusetts Institute of Technology, Cambridge, Massachusetts, United States of America

*Contact: brunner@mit.edu

Abstract: Management of power exhaust will be a crucial task for tokamak fusion reactors. Reactor concepts are often proposed with double-null divertors, i.e., having two magnetic separatrices in an up-down symmetric configuration. This arrangement is potentially advantageous since the majority of the tokamak exhaust power tends to flow to the outer pair of divertor legs at large major radius, where the geometry is favorable for spreading the heat over a large surface area and there is more room for advanced divertor configurations. Despite the importance, there have been relatively few studies of divertor power sharing in near double null configurations and no studies at the poloidal magnetic fields and scrape-off layer power widths anticipated for a reactor. Motivated by this need we have undertaken a systematic study on Alcator C-Mod, examining the effect of magnetic flux balance on the power sharing among the four divertor legs in near double-null plasmas. Ohmic L-modes at three values of plasma current and ICRF-heated enhanced D-alpha (EDA) H-modes and I-modes at a single value of plasma current are explored, producing poloidal magnetic fields of 0.42, 0.62 and 0.85 Tesla. For Ohmic L-modes and ICRF heated EDA H-modes, we find that the point of equal power sharing between upper and lower divertors occurs remarkably close to a balanced double null. Power sharing amongst the outer (upper versus lower) and inner (upper versus lower) pairs of divertors can be described in terms of a logistic function of magnetic flux balance, consistent with heat flux mapping along magnetic field lines to the outer midplane. Power sharing between inner and outer legs is found to follow a Gaussian-like function of magnetic flux balance with non-zero power to the inner divertors at double null. The overall behavior of H-modes operated near double null and for I-modes operating to within one heat flux e-folding of double null are found similar to Ohmic L-modes, with a significant reduction of power on the inner divertor legs. The results are encapsulated in terms of empirically-informed analytic functions of magnetic flux balance. When combined with magnetic equilibrium control system specifications, these relationships can be used to specify the power flux handling requirements for each of the four divertor target plates.

1. Introduction

Power exhaust is an existential challenge for tokamak fusion reactors. Most reactor concepts and burning plasma physics experiments are designed to operate in double null (e.g., UWMAK-I [1], CIT [2], ITER CDA [3], ARIES-I through ARIES-ST and ARIES ACT [4–6], BPX [7], FIRE [8], K-DEMO [9], and ARC [10]). Balanced double null magnetic equilibria have a pair of up-down symmetric poloidal magnetic field nulls located at the same value of poloidal flux, which define the last closed flux surface (LCFS). This configuration has a number of advantages, including: (1) the
majority of the power goes to the divertor legs at large major radius – a result of toroidal geometry combined with enhanced cross-field transport on the low-field side [11]; (2) since the area available to spread the power increases with major radius, the peak heat flux on low field side target plates can be reduced; (3) there is also more physical space to implement advanced divertors in these outer legs; (4) the high-field side scrape-off layer (HFSOL) is magnetically disconnected from the low-field side SOL. Because the HFSOL has low cross-field transport properties, very sharp SOL profiles are produced there in near double-null configurations [11], making it an ideal location for RF actuators – plasma-material interactions on the actuator may be reduced with reduced impurity penetration into the core [12]. Moreover, RF wave physics is highly favorable at this high magnetic field location [13].

Yet, operating sufficiently close to double null to obtain these advantages may be challenging from the control point of view. Such a system must be able to sense deviations from the desired set point and react in time to avoid placing power where it cannot be handled. The SOL cross-field power e-folding, $\lambda_{q}$, is expected to be 1 mm or less in reactor-class devices [14]. For an example of this challenge specific to the Alcator C-Mod control system, we find that a $\sim 2$ mm vertical displacement of the current centroid affects magnetic flux surface mapping of the relative positions of the separatrices to the outer midplane by about $\sim 1$ mm. Taken as a rough guide, this indicates that the plasma vertical position control system must control the plasma current centroid to an accuracy of order $\sim 2\lambda_{q}$ compared to the vertical extent of the confined plasma, $\sim \kappa a$, where $\kappa$ is the plasma elongation and $a$ is the minor radius. While minor radius grows with reactor size, data indicate that $\lambda_{q}$ will remain relatively fixed, set by poloidal magnetic field strength [14]. Consequently, a relative accuracy of order 1 part in 1000 may be needed, depending on reactor size. In addition, the intense radiation environment of a reactor will make measuring the magnetic equilibrium with traditional pick-up coils a challenge [15]. Thus, the first steps in assessing the engineering requirements for plasma position control is to (1) characterize the power sharing of each divertor leg in terms of magnetic flux balance and (2) examine its sensitivity. The principal goal of this paper is to explore Alcator C-Mod data for these dependencies and to identify, if possible, quantitative empirical relationships for them.

Despite the importance of divertor power sharing around double null for reactor designs, there have been very few systematic studies reported. Some of the earliest diverted tokamaks noted dramatic in-out power sharing asymmetries. In double null, thermocouple measurements in PDX identified a 9 to 1 asymmetry in energy deposition, indicating that power flowing from closed to open magnetic surfaces was poloidally asymmetric and concentrated on the low field side [16]. The original ASDEX measured clear in-out asymmetries of line-integrated divertor leg densities as function null balance [17].

DIII-D performed the most extensive exploration of H-mode operation around double null in the early 2000’s. Balances of power and particle fluxes were found to be well represented by a hyperbolic tangent function, parameterized in terms of the distance between the two magnetic separatrices, $\delta R_{sep}$, mapped to the outer mid-plane [18]. A reason for choosing this functional form was not given and fitted parameters were not provided, both of which are the primary goals
of this paper. Heat and particle flux asymmetries were found qualitatively consistent with the influence of poloidal $E \times B$ drifts. In-out peak heat flux asymmetries approaching a factor of 10 [18] were observed, consistent with previous observations of cross-field heat transport being enhanced on low field side SOL. Although the fitted heat flux decay lengths, $\lambda_{HF}$, were found to be insensitive to magnetic balance away from double-null, a somewhat complicated dependence around double null was identified [18]. Stored energy was found to degrade as the $B \times \nabla B$ drift direction went from the primary to secondary divertor [19]. It was found that neutral pumping became relatively ineffective at the inner divertors in double null and that neutral pumping was strongest at the outboard divertor opposite the $B \times \nabla B$ drift direction [20]. Examining the injection of argon into the private flux region, it was found that the radiating divertor was most compatible with a high performance core when the $B \times \nabla B$ drift direction was away from the dominant divertor [21]. EAST has performed examinations of divertor plasma profiles around double null, with results relatively consistent with DIII-D [22].

MAST characterized power flux asymmetries with both Langmuir probes [23] and IR cameras [24,25]. The up-down asymmetry was, like DIII-D, found to follow approximately a hyperbolic tangent function of poloidal flux balance. The in-out energy asymmetry ratio was shown to be much larger during ELMs (1:30) in comparison to between ELMs (1:10). Heat flux widths fitted to the secondary outboard divertor heat flux profile increased as the magnetic equilibrium departed from double null. The outboard divertor heat flux profile exhibited two e-folding lengths, with large fluctuations dominating the second e-folding length region.

The START spherical tokamak used Langmuir probes to measure divertor conditions at double null [26]. Definitive conclusions from the measurements are difficult since only $\sim 1/3$ of the power into the boundary made it to the divertor plates due to losses from high neutral density.

In this paper, we examine divertor heat flux power sharing in Alcator C-Mod obtained from a series of dedicated experiments. Section 2 describes the experimental arrangement. We systematically varied the balance of poloidal flux at the upper-lower x-points on a shot-to-shot basis and swept the strike points across divertor Langmuir probes within a given shot to map out the divertor profiles. Plasma profiles measured at all four divertor target plates are shown in Section 3 for an extensive study of Ohmic L-modes. Section 4 aggregates measurements from the L-mode plasmas with three values of the plasma current. These clearly illustrate the relative balance of power to each divertor as a function of flux balance. Analytic formulations are developed to describe the observed dependence; the data are fit to these model functions and the resulting parameters reported. Dependence on plasma current is explored. Section 5 examines divertor conditions near double null for EDA H-mode and I-mode confinement regimes. Fitted parameters for EDA H-modes are reported. Section 6 presents a concise summary of the principal findings.

2. Experimental arrangement

These experiments were performed in Alcator C-Mod [27], a compact (major radius $R=0.67$ m) high-field (toroidal magnetic field $B_T<8$ T and poloidal magnetic field $B_p<1.4$T) tokamak. Divertor
conditions were explored for Ohmic L-mode plasmas at plasma currents of 0.55 MA, 0.8 MA, and 1.1 MA with line-averaged densities of 0.55x10^{20} m^{-3}, 0.67x10^{20} m^{-3}, 1x10^{20} m^{-3} respectively, maintaining them all at approximately the same Greenwald fraction, \( f_G \approx 0.14 \). Data were also taken for EDA H-mode and I-mode plasmas at the single plasma current of 0.8 MA and with line-averaged densities of 2.5x10^{20} m^{-3} and 0.9x10^{20} m^{-3} respectively. Approximately 2 MW of ICRF heating were applied for these high confinement plasmas. All data were taken at a toroidal field of 5.4 T. L-mode and H-mode plasmas had the \( \mathbf{B} \times \nabla B \) direction pointed \textit{towards} the lower x-point and I-mode plasmas had the \( \mathbf{B} \times \nabla B \) direction pointed \textit{away} from the lower x-point.

C-Mod has extensive arrays of divertor Langmuir probes at all four divertor locations (Figure 1). By utilizing a small sweep of the plasma strike points through the course of a plasma shot, detailed measurements of the plasma conditions across the divertor targets were obtained. The lower-outer divertor plate is a vertical target with a relatively closed geometry. The inner-lower divertor plate is a vertical target with a somewhat less closed geometry. The upper divertor is a flat plate with both the inner and outer divertors joined as one continuous structure. There is a cryopump behind the upper divertor with slots on the outboard side [28]. It was not used for these experiments. Experiments were performed under low recycling conditions where volumetric dissipation in the divertor was negligible with respect to power balance. In this

Figure 1 - Example of an intra-shot sweep of the plasma equilibrium over the divertor Langmuir probes (green circles). Orange profile is the LCFS at the start of the sweep and purple profile is the LCFS at the end of the sweep. This is a lower null equilibrium with \( \delta R_{sep} \approx -3 \text{ mm} \). Each of the four divertors are indicated with their labels and color codes used through this paper.
situation, the heat flux footprint measured at the divertor target can be taken as indicative of the ‘upstream’ plasma heat flux profile when magnetically mapped to the outer midplane.

Null balance is quantified by $\delta R_{\text{sep}}$, the distance between the two magnetic separatrices mapped along magnetic flux surfaces to the outer mid-plane (sometimes also called $S_{\text{sep}}$ for the position of the secondary separatrix relative to the primary). Negative $\delta R_{\text{sep}}$ corresponds to lower null plasmas, positive $\delta R_{\text{sep}}$ corresponds to upper null plasmas, and $\delta R_{\text{sep}} = 0$ is double null. An example of each configuration is shown in Figure 2. The primary divertor is the one that is directly connected to the LCFS whereas the secondary divertor is not directly mapped to the LCFS. In lower null the lower divertors are primary and the upper divertors are secondary.

![Figure 2 - Equilibrium, last closed flux surface, and secondary separatrix for lower null, double null, and upper null magnetic topologies.](image)

Within a given shot, the locations of the strike points were swept across the divertor surfaces and divertor probe arrays, as shown in Figure 1, while maintaining $\delta R_{\text{sep}}$ relatively constant. We label poloidal flux surfaces with the symbol $\rho$, which corresponds to the radial distance into the SOL of that same poloidal flux value at outer mid-plane. The strike-point sweeps were designed to produce well defined ‘trajectories’ of probe data in $\rho$ space. The vertical position of plasma current centroid ($Z_{\text{cur}}$) was scanned shot-to-shot to scan $\delta R_{\text{sep}}$ with an approximate step size of ~1.5 mm. A complete data set of divertor conditions versus $\rho$ and $\delta R_{\text{sep}}$ for a given core condition was attained in 5 shots. These experiments spanned $\delta R_{\text{sep}} \approx \pm 4$ mm, which was larger than one SOL power e-folding distance for all cases studied. Although the programmed values of $\delta R_{\text{sep}}$ were fixed, there was inter-shot variability in value of $\delta R_{\text{sep}} \approx \pm 1$ mm. Consequently, the data obtained largely filled in the targeted $(\rho, \delta R_{\text{sep}})$ space between set-point values of $\delta R_{\text{sep}}$.

3. Heat flux profiles in L-mode
Divertor Langmuir probe measurements of plasma density \(n_e\), electron temperature \(T_e\), ground current density \(J_{\text{gnd}}\), and heat flux density parallel to the magnetic field \(q_{||}\) for the L-mode current cases are shown in Figures 3-5. Ground current density is the current density arriving at the probe when it is biased at the potential of the divertor, i.e., zero volts in this case. The parallel heat flux density is calculated using standard sheath theory including effects of finite current through the sheath [29]:

\[
q_{||} = \gamma T_e J_{\text{sat}},
\]

\[
\gamma = 2.5 \frac{T_i}{Z T_e} + 2 \frac{f_{\text{gnd}}}{1-\delta_e} - \frac{1}{2} \ln \left[ 2\pi \frac{m_e}{m_i} \left( Z + \frac{T_i}{T_e} \right) \left( \frac{1-f_{\text{gnd}}}{1-\delta_e} \right) \right],
\]

where \(\gamma\) is the sheath heat flux transmission coefficient and \(J_{\text{sat}}\) is the ion saturation current. The ion saturation current is the current collected by the probe divided by probe surface area projected parallel to the incident magnetic field. \(T_i\) is the ion temperature, which is assumed to be equal to the electron temperature, which is a good approximation in the collisional C-Mod divertor [30]. \(Z\) is the ion charge and is assumed to be 1 (deuterium). \(\delta_e\) is the secondary electron emission coefficient, which is taken to be zero due to the shallow angle of incidence causing prompt recapture of secondary electrons and the relatively modest (<50 eV) incident electron temperature causing a low secondary emission [31]. The ratio of electron to ion mass assumes deuterium plasma. The plasma density is related to the ion saturation current through: 

\[
n_e = J_{\text{sat}} / \left( e \sqrt{(Z T_e + T_i) / m_i} \right).
\]

The profiles in Figures 3-5 are averaged over multiple sweeps to reduce the effects of plasma fluctuations. Once can consider the scatter in the datapoints shown in Figures 3-5 as a measure of the error bar due to plasma fluctuations. But, in addition to this, one should allow for systematic errors on the order of \(\pm 20\%\) for the ordinate values of these graphs. Although magnetic equilibrium mapping of multiple diagnostics typically shows systematic errors on the order of a few millimeters, these divertor probe profiles are seen to be consistent with magnetic mapping to sub-millimeter precision. However, one should allow for possible systematic errors on the order of \(\sim 1\) mm in the abscissa of these graphs.

The profiles have the following salient characteristics: The inner divertor density profiles are relatively symmetric about the strike point. The outer divertor density profiles are very asymmetric, with a steep fall off in the private flux region and a much broader profile into the common flux region. The density scale lengths in the upper-outer divertor common flux zone is wider than for the lower-outer. There appears to be some pressure \(p = 2n_e T_e\) loss at the inner divertors relative to the outer divertors while they are magnetically mapped, especially at low plasma current, indicating that there may have been some volumetric dissipation at the inner divertors. For all of the divertors, the part of the density profile that should map magnetically to the outer midplane inside the secondary separatrix remains relatively unchanged during the \(\delta R_{\text{sep}}\) scan whereas, as expected, the part that should become magnetically disconnected (see Figure 2) appears to do so at a value of \(\rho\) value corresponding to \(|\delta R_{\text{sep}}|\).
Figure 3 - Divertor plasma profiles measured by Langmuir probes, mapped to the distance beyond the LCFS at the outer midplane according to poloidal magnetic flux. These data are from 0.55 MA, L-mode plasmas with different values of $\delta R_{sep}$. Profiles are color-coded for each value of $\delta R_{sep}$ and vertical lines are added to each plot at the $\rho$-location of $\delta R_{sep}$ for a given $\delta R_{sep}$. Top row is density, second row electron temperature, third row ground current density, fourth row sheath heat flux density. Columns left to right correspond to inner lower divertor, outer lower divertor, outer upper divertor, and inner upper divertor.
Figure 4 - Same as Figure 3, but for 0.8 MA plasmas.
Figure 5 - Same as Figure 4, but for 1.1 MA plasmas.
The electron temperature profiles follow most of the same trends as the density profile, except that they exhibit a two e-folding profile in the common flux region. A narrow channel exists near the strike point where most of the heat flux is carried to the divertor plate. The inner divertor channels are narrower than the outer. The second e-foldings on the outer divertor profiles result in relatively flat electron temperature profiles.

The ground current profiles are included because they provide an important contribution to the sheath heat flux (Equation 1), being comparable in magnitude to the ion saturation current. Positive current corresponds to net ion current collected by the plate. The ground current is comprised of two main components, thermoelectric [32] and Pfirsch-Schlüter [33], which have been studied extensively in C-Mod [34]. Thermoelectric currents result from different electron temperatures at the ends of each flux tube setting a different sheath potential drop. Because the inner and outer divertor targets are electrically connected at the same potential, the potential difference drives a current between the sheaths. They typically result in moderate, broad ground current profiles in the common SOL with opposite polarities at inner versus outer divertors. On closed flux surfaces, Pfirsch-Schlüter currents arise to satisfy MHD equilibrium in a toroidal geometry. On open flux surfaces, they can flow into the electrically conducting divertor target plates. They typically appear as narrow peaks of current that change sign across the strike point a divertor plate. As a result, the zero-crossing point in the profile can be a good marker for the strike point [35].

The parallel heat flux profiles largely follow the same story as the temperature and density profiles. However, the two e-folding characteristics in the common flux zone is slightly less prevalent in the parallel heat flux profiles than in the electron temperature profiles.

4. L-mode divertor power sharing and analytic fitting functions

Taking the above data in aggregate, we now examine the details of power sharing among the divertors with respect to $\delta R_{sep}$, Figure 6. The Langmuir probe parallel heat flux data at the divertor target ($q_{ll,div}$) are first mapped to the outer midplane, accounting for the magnetic field strength at the measurement location compared to the outer midplane, $q_{ll,mid} = q_{ll,div}B_{mid}/B_{div}$, which conserves heat flow along a magnetic flux tube. These data are separated into $\delta R_{sep}$ bins, sorted by $\rho$, and integrated across $\rho$ to compute a power measurement for each divertor surface at each $\delta R_{sep}$ bin. The divertor integrated power measurements are then combined and normalized with respect to total power for each $\delta R_{sep}$ bin.

From the data shown in Figure 6, we observe that: (1) The $\delta R_{sep}$ value for which equal power sharing amongst the upper and lower divertors is close to the point of balanced double null (<0.5 mm), as inferred from the magnetics. This is a remarkable result, suggesting that the magnetic reconstruction of the boundary can be trusted to a high precision. (2) The inner divertors have very little of the total power around double null (~5% for each). (3) $|\delta R_{sep}|$ must exceed ~2 mm away from double null ($\sim \lambda_q$) before the inner divertors see a large increase in the fraction of total power. (4) The slopes of the power sharing curves increase with increasing plasma current,
as expected from a narrowing of the SOL power width at higher poloidal magnetic field. (5) Away from double null the ratio of the power from the outer to the inner divertors is ~2:1 and greater at lower plasma current.

![Graph showing fraction of total power flux to divertors as a function of \(\delta R_{sep}\).](image)

**Figure 6 - Balance of total power to four divertors as a function of \(\delta R_{sep}\).** Results are shown from Ohmic L-mode plasmas at three plasma currents, as indicated.

We now develop analytic models to track these empirical observations, guided by a simplified picture of boundary heat transport. Three different observations are considered: (1) sharing of power between upper-outer and lower-outer divertor targets; (2) sharing of power between upper-inner and lower-inner divertor targets; (3) fraction of total exhaust power on inner and outer divertor targets. Data taken from three different plasma currents are used to assess reproducibility and to look for dependence on poloidal magnetic field.

### 4.1 Sharing of power between upper-outer and lower-outer divertor targets versus \(\delta R_{sep}\)

A simple picture that describes the up-down power sharing on the outer divertors is assembled as follows. Experimental observations indicate that most of the power that crosses into the boundary plasma does so at the outer midplane. For the purpose of this power sharing calculation, it is assumed that the resultant heat flux profile is exponential with a single e-folding width, \(\lambda_0\). This assumption is justified for the L-mode cases examined here because the power flux due to the second e-folding length is at least an order of magnitude lower than the primary e-folding and thus contributes negligibly to this analysis. Each of the outer divertors receives the fraction of the power flux profile that maps magnetically to that divertor. From this simple picture, the ratio of the power flowing to the outer lower divertor to the total power to both outer divertors can be constructed from the following relationships:
\[ P_{ol} \propto \left\{ \begin{array}{ll}
\int_{0}^{\infty} e^{-\rho/\lambda_o} d\rho, & \delta R_{sep} < 0 \\
\int_{|\delta R_{sep}|}^{\infty} e^{-\rho/\lambda_o} d\rho, & \delta R_{sep} \geq 0
\end{array} \right. \]

\[ P_{ou} \propto \left\{ \begin{array}{ll}
\int_{0}^{\infty} e^{-\rho/\lambda_o} d\rho, & \delta R_{sep} < 0 \\
\int_{|\delta R_{sep}|}^{\infty} e^{-\rho/\lambda_o} d\rho, & \delta R_{sep} \geq 0
\end{array} \right. \]

leading to logistic functions of \( \delta R_{sep}/\lambda_o \),

\[ \frac{P_{ol}}{P_{ol}+P_{ou}} = \frac{1}{1+e^{\delta R_{sep}/\lambda_o}} \quad \text{and} \quad \frac{P_{ou}}{P_{ol}+P_{ou}} = \frac{1}{1+e^{-\delta R_{sep}/\lambda_o}}. \quad (2) \]

Taking the difference in these ratios results in a hyperbolic tangent relationship, as was found in the DIII-D data [18]:

\[ \frac{P_{ol}-P_{ou}}{P_{ol}+P_{ou}} = \frac{1-e^{\delta R_{sep}/\lambda_o}}{1+e^{\delta R_{sep}/\lambda_o}} = \tanh \left( \frac{-\delta R_{sep}}{2\lambda_o} \right). \quad (3) \]

Note factor 2 difference in front of the e-folding between the logistic and hyperbolic tangent formulations.

4.2 Sharing of power between upper-inner and lower-inner divertor targets versus \( \delta R_{sep} \)

In considering a simple poloidal flux mapping picture like the outer divertors, a simple analytic relationship for the up-down power share of the inner divertors is not readily identified. Guided by the experimental data presented here, we note that the power sharing appears to follow a similar logistic function trend but with a distinctly different e-folding length (\( \lambda_1 \)). The power to the inner lower divertor divided by the sum of power to both inner divertors thus can be parameterized as

\[ \frac{P_{il}}{P_{il}+P_{iu}} = \frac{1}{1+e^{\delta R_{sep}/\lambda_1}}. \quad (4) \]

4.3 Fraction of total exhaust power on inner and outer divertor targets versus \( \delta R_{sep} \)

Three key observations must be considered in developing an analytic function to describe how the fraction of the total exhaust power arriving on inner divertor targets varies with \( \delta R_{sep} \): (1)
the relative power fluxes between the inner and outer divertors are typically unbalanced, even far away from double null; (2) at double null, a non-negligible fraction (~10%) of the total power exhaust flows to the inner divertors, despite being magnetically disconnected from the outer SOL; (3) the shape of the in-out power sharing versus $\delta R_{\text{sep}}$ is found to not follow a logistic relationship. Experimental measurements shown here suggest that a Gaussian-like dependence on $\delta R_{\text{sep}}$ is more appropriate. Taking these considerations into account, the flowing analytic relation for the fraction of total exhaust power arriving on inner and outer divertor targets versus $\delta R_{\text{sep}}$ is proposed:

$$\frac{P_{\text{ii}}+P_{\text{io}}}{P_{\text{ii}}+P_{\text{io}}+P_{\text{ol}}+P_{\text{ou}}} = P_{1,0} + (P_{1,0} - P_{1,\infty}) \left(1 - \frac{2}{1+e^{-\left(\delta R_{\text{sep}}/\lambda_{\text{io}}\right)^2}}\right),$$

where $P_{1,0}$ is the fraction of the power to the inner divertors at $\delta R_{\text{sep}} = 0$, $P_{1,\infty}$ is the fraction of the power to the inner divertors at $\delta R_{\text{sep}} = \pm \infty$, and $\lambda_{\text{io}}$ is the characteristic Gaussian scale length.

4.4 Results from fitting divertor power sharing data to model functions

The analytical functions identified in Eqs. (2), (4) and (5) form a complete characterization of the power sharing among the four divertor surfaces; an appropriate combination of these can be used to determine the fraction of total exhaust power arriving at any one divertor surface for a given value of $\delta R_{\text{sep}}$. The free parameters that must be determined from experiment are: $\lambda_o$, $\lambda_i$, $\lambda_{\text{io}}$, $P_{1,0}$, and $P_{1,\infty}$. In addition, we must allow for uncertainty in identifying the location of the separatrix flux surface ($\rho = 0$) as it strikes the divertor target. To account for this, we allow each of the logistic and Gaussian fits to be translated linearly in $\delta R_{\text{sep}}$ by a quantity $\rho_{0,x}$, and determine that value by fitting the data (i.e., $\delta R_{\text{sep,used in function}} = \delta R_{\text{sep}} - \rho_{0,x}$). Thus, a complete description of the experimental observations for a given plasma current can be encapsulated in fit 8 parameters, $\lambda_o$, $\lambda_i$, $\lambda_{\text{io}}$, $P_{1,0}$, $P_{1,\infty}$, $\rho_{0,o}$, $\rho_{0,i}$, and $\rho_{0,io}$, with the subscripts (o, i, io) referring to formulas for outer divertor (Eq. 2), inner divertor (Eq. 4), and inner-outer power sharing (Eq. 5).

The results of fitting these three functions to the divertor power measurements made at three different plasma currents are shown in Figure 7. The fitted parameters and their dependence on poloidal magnetic field are show in Figures 8-10. We postulate a power law dependence on poloidal magnetic field for the fitted parameters of the form $F(B_p) = A B_p^k$. Numerical values for the fit parameters and their corresponding exponent (k) are shown in Table 1.
The logistic fits to both the inner and outer up-down power ratios match the data quite well. This simple model is clearly adequate to describe this power flux ratio around double null. The e-foldings display the characteristic inverse dependence on plasma current (or equivalently poloidal magnetic field, for typical diverted plasmas in C-Mod $B_p [\Gamma] \approx 0.77 \times I_p [\text{MA}]).$ The inner
divertor e-folding appears to have a slightly stronger than inverse scaling and the outer divertor a slightly weaker. However, due to the small amount of data within this scan we do not consider this trend to be significant. The e-folding for the outer divertor is consistently \(~3\) times larger than the inner divertor. This is consistent with the narrower inner divertor heat flux profiles shown in Figures 3-5 as well as upstream measurements [36]. The inner up-down ratio is shifted to negative \(\delta R_{\text{sep}}\) whereas the outer up-down ratio is shifted to positive \(\delta R_{\text{sep}}\). This is consistent with poloidal \(E_T \times B\) heat convection transporting heat from the inner lower divertor to the inner upper divertor and from the outer upper divertor to the outer lower divertor [11]. The shifts have very similar values between the inner and outer divertors. And they decay to lower values at higher poloidal magnetic fields. The fact that these shifts are relatively small indicates that all drifts, \(E_T \times B\) and \(B \times \nabla B\), do not play a dominant role in power sharing.

The story for the Gaussian-like fit to the fraction of total power on the inner versus outer divertor is not as satisfying. The examination would benefit from a wider scan in \(\delta R_{\text{sep}}\), especially at low poloidal field where the e-foldings are long. At the minimum near double null the fraction of the total power flux to the inner divertors is consistently \(~5\)% across the poloidal field scan. Far away from double null the fraction of the total power flux to the inner divertors is \(~20\)% to \(25\)% with perhaps a slight increase with poloidal magnetic field. The offset in \(\delta R_{\text{sep}}\) is essentially zero for the larger poloidal magnetic field cases, indicating perhaps very accurate magnetic reconstructions of the plasma equilibrium. The offset at the lowest field is much larger. However, the very small amount of data from the inner divertor and the limited scan of \(\delta R_{\text{sep}}\) with respect to the SOL width lend low confidence to the precise value of this data point. The e-foldings for the Gaussian-like fit are much closer to the e-foldings obtained from the fit to the ratio of powers on the outer divertors than for inner divertors.

The story for the Gaussian-like fit to the fraction of total power on the inner versus outer divertor is not as satisfying. The examination would benefit from a wider scan in \(\delta R_{\text{sep}}\), especially at low poloidal field where the e-foldings are long. At the minimum near double null the fraction of the total power flux to the inner divertors is consistently \(~5\)% across the poloidal field scan. Far away from double null the fraction of the total power flux to the inner divertors is \(~20\)% to \(25\)% with perhaps a slight increase with poloidal magnetic field. The offset in \(\delta R_{\text{sep}}\) is essentially zero for the larger poloidal magnetic field cases, indicating perhaps very accurate magnetic reconstructions of the plasma equilibrium. The offset at the lowest field is much larger. However, the very small amount of data from the inner divertor and the limited scan of \(\delta R_{\text{sep}}\) with respect to the SOL width lend low confidence to the precise value of this data point. The e-foldings for the Gaussian-like fit are much closer to the e-foldings obtained from the fit to the ratio of powers on the outer divertors than for inner divertors.

![Figure 8](image)

Figure 8 – Scaling of analytic parameters to the outer lower-upper logistic fit (Eq. 2) in Figure 7 with poloidal magnetic field. Yellow points are parameters from fits shown in Figure 7 with vertical bars showing the 1 standard deviation error. Green lines are the results of power law fits to the data with the lighter green band showing the 1 standard deviation error of the power law fits.
Figure 9 – Similar to Figure 8 but now the scaling of analytic parameters to the in-out Gaussian fit (Eq. 5).

Figure 10 – Similar to Figure 8 but now the scaling of analytic parameters to the inner lower-upper logistic fit (Eq. 4).
Table 1 - Poloidal field power law dependence of analytic function fit parameters, as shown in Figures 8-10.

<table>
<thead>
<tr>
<th>Model function: $F(R_p) = A B^\frac{\rho}{\lambda}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer div. up-down sharing (Eq. 2)</td>
</tr>
<tr>
<td>Inner div. up-down sharing (Eq. 4)</td>
</tr>
<tr>
<td>Inner divertor total power fraction (Eq. 5)</td>
</tr>
<tr>
<td>$\lambda_0$</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>±0.2 [mm/T]</td>
</tr>
<tr>
<td>k</td>
</tr>
</tbody>
</table>

4.5 Results from fitting heat flux profiles with Eich model

An additional way of analytically quantifying the power balance among the divertors is to fit the divertor heat flux profiles with an analytic function and observe how the fitting parameters vary with $\delta R_{sep}$. The ‘Eich’ equation is a common one used to parameterize boundary profiles [14]. It is the result of convolving an exponential profile (truncated at $\rho = 0$ and e-folding $\lambda_q$) and a Gaussian (width $S$):

$$q_{\parallel} (\rho) = q_0 \frac{\exp \left( \left( \frac{S}{2\lambda_q} \right)^2 - \frac{\rho}{\lambda_q} \right)}{\exp \left( \frac{S}{2\lambda_q} \right)} \text{erfc} \left( \frac{S}{2\lambda_q} - \frac{\rho}{S} \right).$$

As with the functions fitted to the power ratios, we allow this to be shifted linearly in $\rho$ and determine this free parameter $\rho_0$, by fitting the data. Here we omit the background heat flux term since the measurements are not with thermal sensors (e.g., IR or surface thermocouples) but plasma flux and thus do not see a ‘background’ heat flux due to radiation or neutral particles.

The results of fitting the ‘Eich’ profiles are shown in Figure 11. The fitted peak power flux ($q_0$) behaves as expected, highest when a divertor is primary and decaying away as the divertor becomes secondary. The fitted spreading factor ($S$) has error bars up to the same order as the parameter. It neither has a clear trend with $\delta R_{sep}$ nor has systematic differences between divertors. The spreading parameter is often considered representative of cross-field dissipation processes that are dependent on divertor geometry. However, we observe no clear differences amongst the divertors despite three very different divertor geometries, although these measurements were under low dissipation conditions.

The fitted e-foldings ($\lambda_q$) have differences among divertors and different trends with $\delta R_{sep}$ and plasma current. Consistent with the logistic fits, the inner divertors have systematically smaller $\lambda_q$ than the outer divertors. The inner divertors have no clear trend with $\delta R_{sep}$ across the plasma current scan. However, as the magnitude of $\delta R_{sep}$ gets larger, the fitted $\lambda_q$ for the secondary outer divertor gets larger. Examining Figures 3-5, we see that this trend is not due to an actual increase in the near-SOL e-folding, but is instead due to the secondary far-SOL e-folding dominating the fit. A similar trend was seen in MAST [24].

Finally, the shift parameter ($\rho_0$) is relatively constant around $\delta R_{sep} = 0$ for the primary divertors and increases approximately linearly with $\delta R_{sep}$ for the secondary divertors. These trends are as expected. For the primary divertors, the location of the peak heat flux in absence of spreading is at the strike point ($\rho_0 = 0$). For the secondary divertors, this location follows the location of the secondary strike point ($\rho_0 = |\delta R_{sep}|$).

**4.6 Evidence for non-Gaussian spreading of heat flux into private zone**

Langmuir probe inferred heat flux profile data shown in Figures 3-5 have a very large dynamic range across the divertor targets, approximately 5 orders of magnitude. This is because the sensitivity of the electronics for each probe is adjusted according to the local ion saturation current values. This dynamic range exceeds that of thermal systems (IR or surface thermocouples) by ~1-2 orders of magnitude. Consequently, the Langmuir probes can reveal important features beyond the detection limits of thermal sensors, such as the heat flux profile in the private flux region.
We note that the ‘Eich’ heat flux fitting function assumes that a diffusive spreading of heat occurs across the profile, which is solely responsible for filling in the private flux zone. However, if this assumption were true, it should produce an inverted parabolic profile for the private flux region on the log-linear plots shown in Figures 3-5. Yet the heat flux profiles are seen to be approximately straight lines on these plots. These data indicate that the profiles are basically exponential on either side of the strike-point with perhaps some Gaussian spreading. The model function should therefore include an exponential fall-off term for the private flux region, rather than rely on a Gaussian spreading model alone. We also note that the second e-folding of the far SOL is clearly not included in the Eich model function. If this feature is not properly resolved by the heat flux diagnostic (e.g. sensitivity too low) it could be mistakenly interpreted as part of the uniform background component of the model function. A more accurate analytic description of the measured heat flux profile:

\[ q_{\parallel}(\rho) = \begin{cases} 
q_0 e^{\rho/\lambda_{q,p}}, & \rho < 0 \\
(q_0 - q_{cl}) e^{-\rho/\lambda_{q,cl}} + q_{cl} e^{-\rho/\lambda_{q,cf}}, & \rho \geq 0 .
\end{cases} \]  

(7)

Where \( (\lambda_{q,cl}, \lambda_{q,cf}, \lambda_{q,p}) \) are the characteristic heat flux lengths in the common flux near and far regions and the private flux region, respectively. The peak heat flux is \( q_0 \) and the contribution of the heat flux in the far common flux region is \( q_{cl} \). These features will be examined in more detail in a future paper examining a large database of heat flux width scalings in C-Mod.

5. EDA H-mode and I-mode divertor power sharing

Divertor power sharing as a function of magnetic flux balance (\( \delta R_{\text{sep}} \)) was also studied for two stationary, high-confinement regimes: EDA H-mode and I-mode.

The Enhanced D-Alpha (EDA) H-mode [37] has both a particle and heat flux transport barrier, as evidenced by the formation of density and temperature pedestals, but lacks the cyclic relaxation of Type I ELMs (edge-localized modes) typical of H-modes in many experiments [38]. C-Mod can operate with an ELMy H-mode [39]; however, it requires a magnetic equilibrium that places the outer lower strike point away from the divertor Langmuir probe array. EDA H-modes were produced with the \( B \times V B \) direction pointed towards the lower x-point (‘forward’ field for C-Mod), similar to the L-mode plasmas reported above. In lower null, this is called the ‘favorable’ drift direction due to the lower threshold power for access to H-mode.

The I-mode confinement regime [40] has an energy transport barrier but lacks a density barrier, as evidenced by the formation of only a temperature pedestal. It also lacks ELMs. The I-mode experiments were performed with the \( B \times V B \) direction pointed away from the lower x-point (‘reversed’ field for C-Mod). This ‘unfavorable’ drift direction (in lower null) allows for a larger power window with access to I-mode. I-mode has typically been operated in single null with \( |\delta R_{\text{sep}}| \sim 10 - 20 \) mm. In these strongly single-null divertor cases, an in-out power asymmetry of \( \sim 2:1 \) has been detected previously [41], which is reverse to what is typically seen for H-modes. The increased power to the inner divertor in I-mode could be an additional challenge for power
exhaust solutions. Operation in a near-double null configuration is thought to be a way to mitigate the inner divertor power load. We report here some of the first experiments attempting to operate I-mode in near double null, see also Ref. [42].

5.1 Divertor power sharing in EDA H-mode versus $\delta R_{\text{sep}}$

A series of 4 EDA H-mode plasmas were produced, scanning $\delta R_{\text{sep}}$ over the range of $-4 \text{ mm} < \delta R_{\text{sep}} < 4 \text{ mm}$. Core plasma conditions were held fixed: toroidal field of 5.4 T, plasma current of 0.8 MA, line-averaged density of $2.5 \times 10^{20} \text{ m}^{-3}$ and ICRF heating at approximately 2 MW. For each plasma, H-mode was initiated with $\delta R_{\text{sep}} < -4 \text{ mm}$ and then $\delta R_{\text{sep}}$ was quickly ramped to the desired value. When $\delta R_{\text{sep}}$ was pushed beyond $\sim 3 \text{ mm}$ into the ‘unfavorable’ drift direction, core confinement started to degrade and the H-mode would back transition to an L-mode. These plasmas had $P_{\text{in}}/P_{\text{L-H}} \approx 1.4$, which was apparently insufficient to avoid H-L back transitions for the ‘unfavorable’ drift configuration.

Divertor power flux sharing in H-mode, Figure 12, is found to be similar to that for L-mode but with a few key differences. The minimum power going to the inner divertor near double-null was $\sim 3$-4 times higher in H-mode than L-mode ($\sim 15$-$20\%$ compared to $\sim 5\%$). Also, the outer divertor up-down power ratio saturated at $\sim 3$:$1$ away from double null. In these H-modes the secondary e-folding length is much longer than the primary. Consequently, the outer divertor second e-folding contributes a much larger fraction of the total power flux to each divertor. Compared to L-mode plasmas at the same current, the equilibrium must be pushed further away from double null in order to achieve the same ratio of power flux reduction to the secondary divertor.
Following a similar procedure to that employed for the L-modes, we fitted analytic functions to the divertor power sharing. The fitted parameters are shown in Table 2. Equations 4 (logistic) and 5 (Gaussian) were found to be appropriate functions to describe the inner divertor up-down power sharing and inner divertor power fractions. The logistic fit function for outer divertor up-down power sharing (Equation 2) had to be modified to account for very broad secondary e-folding lengths,

\[
\frac{P_{oi}}{P_{oi}+P_{ou}} = \frac{P_{\infty}^{-1}}{1+e^{\delta R_{sep}/\lambda_0}} + 1 - P_{\infty},
\]

where \( P_{\infty} \) is the saturated level far away from \( \delta R_{sep} = 0 \). The resulting fitted analytic functions are shown as yellow curves in Figure 13.

### Table 2 - Fitted parameters for divertor power sharing in H-mode.

<table>
<thead>
<tr>
<th>Gaussian-like fit for inner divertor power fraction (Eq. 5)</th>
<th>Logistic fit for inner div. up-down sharing (Eq. 4)</th>
<th>Modified logistic fit for outer div. up-down sharing (Eq. 7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{i,0} )</td>
<td>( P_{i,\infty} )</td>
<td>( \lambda_i [\text{mm}] )</td>
</tr>
<tr>
<td>0.16 ± 0.03</td>
<td>0.41 ± 0.03</td>
<td>1.57 ± 0.31</td>
</tr>
</tbody>
</table>

Figure 13 - Divertor heat flux power sharing for H-mode. Yellow curves are the analytic functions of Equations 5, 4 and 7 evaluated using the fitted parameters shown in Table 2.

5.2 Divertor power sharing in I-mode versus \( \delta R_{sep} \)
I-mode core plasma conditions were: toroidal field of 5.4 T, plasma current of 0.8 MA, line-averaged density of 0.9x10^{20} m^{-3} and ICRF heating at approximately 2 MW. The I-mode plasmas, like the H-mode, were initiated with $\delta R_{sep} < -4$ mm and then $\delta R_{sep}$ was quickly ramped to the desired position. However, only a partial scan of $\delta R_{sep}$ from -4 to -1 mm was possible. For larger values of $\delta R_{sep}$ the plasma would transition out of I-mode to H-mode. We have found in some instances that sweeping $\delta R_{sep}$ in an I-mode to the ‘favorable’ drift direction causes a transition to an H-mode with exceptionally high confinement ($H_{98} \sim 2$), these plasmas are thought to be accessing the “super H-mode” branch [43,44]. Unfortunately, these tend to be of short duration and are not stationary due to a lack of an edge relaxation mechanism capable of preventing core impurity accumulation.

The limited range of $\delta R_{sep}$ for the I-modes does not allow fitting to the analytic functions. However, three important conclusions can be made from these data: (1) I-mode can be operated within $\sim \lambda_q$ of double null; (2) the in-out ratio of power flux between the primary divertors is reduced compared to what is typically seen in single null I-modes ($\delta R_{sep} = -10$ to -20 mm) from between 67% and 75% to between 5% and 25%, depending on $\delta R_{sep}$ – good news for reducing the inner divertor power handling challenge; (3) I-mode operation near double null helps with the outer divertor power load as well, with the secondary outer divertor taking up to $\sim 30\%$ of the power flux.

Figure 14 - Divertor power sharing for 0.8 MA I-mode plasmas.
6. Summary and Conclusions

We have performed a systematic, detailed study of power sharing amongst four divertor targets in Alcator C-Mod (outer-lower, inner-lower, outer-upper, inner-upper) and its dependence on poloidal magnetic flux balance in double-null plasmas as parameterized by $\delta R_{sep}$. L-mode plasmas at three plasma currents (0.55, 0.8, 1.1 MA) are studied as well as I-mode and EDA H-mode plasmas at fixed plasma current (0.8 MA). All data were taken with a toroidal field of 5.4 Tesla. The corresponding poloidal magnetic fields were 0.42, 0.62 and 0.85 Tesla. The goals of this research were to: (1) assemble power sharing data at high poloidal magnetic fields accessible to C-Mod, (2) examine systematic trends with regard to poloidal magnetic field dependence and (3) assemble fitted empirical formulas that encapsulate the power sharing results to help inform requirements for magnetic flux balance control and divertor power loads for future high-power density devices.

The research led to the following principal findings:

1. The up-down power sharing for both of the pairs of inner and outer divertors are found to follow a simple logistic function of $\delta R_{sep}$ when the SOL heat flux is dominated by the short e-folding length associated with the near SOL, as in the case of L-modes and I-modes; this behavior is expected through a simple magnetic mapping of the ‘upstream’ heat flux. When there is significant power in the second e-folding length of the far SOL, such as in EDA H-modes, this picture becomes more complicated but still follows the picture of divertor heat flux mapping along magnetic flux surfaces to ‘upstream’ profiles.

2. The e-folding lengths (logistic profiles) associated with the outer divertor are ~3 times larger than those for the inner divertor in L-mode.

3. The e-folding lengths (logistic profiles) are found to have an approximately inverse dependence on poloidal magnetic field, consistent with the general trend of heat flux profiles seen elsewhere.

4. The point of equal power sharing amongst inner and outer divertor pairs is offset in $\delta R_{sep}$ by less than one heat flux e-folding in a direction that is consistent with poloidal $E_B \times B$ heat convection. This offset is likely to be reduced as divertor dissipation is increased [11].

5. The in-out ratio of power flux follows a Gaussian-like function of $\delta R_{sep}$. The physical origin of this dependence is not yet clear.

6. The e-folding length of the Gaussian-like profile is more comparable to the outer divertor e-folding (logistic profile) than to the inner divertor e-folding (logistic profile) in L-mode.

7. At double null ($\delta R_{sep} = 0$) the fraction of the total power flux to the inner divertors is ~5% for L-mode, <20% for H-mode, and <10% for I-mode. This is a non-negligible fraction of the total power and must be taken into account for machines designed to operate at double null.
8. Away from double null the fraction of total power flux to the inner divertors increases to ~25% for L- and I-mode and ~40% for H-mode.

9. The ‘Eich’ heat flux profile does a satisfactory job of fitting the peak and main e-folding lengths of the observed divertor heat flux profiles but is lacking features to match the heat flux profiles observed in the private regions for both the inner and outer divertors as well as the far SOL secondary e-folding lengths observed in the outer divertor profiles.

10. The heat flux peak and offset in the ‘Eich’ profile behave as expected with $\delta R_{\text{sep}}$.

11. The spreading parameter, as inferred from the Eich profile, does not exhibit any trend with respect to $\delta R_{\text{sep}}$ or divertor geometry. However, secondary e-folding lengths not included in the Eich function can complicate the picture.

12. High dynamic range measurements of heat flux in private flux regions reveal an exponential profile shape, spanning over three orders of magnitude. These data indicate that the process that ‘fills in’ the private zone cannot be simply described in terms of a symmetric, Gaussian spreading function, as included in the Eich model. Instead, these data suggest that cross-field transport process is fundamentally asymmetric, leading to a large e-folding length on the low-field side and a small e-folding length on the high field side of the strike point location. Such an asymmetry in cross-field transport may be due to curvature-driven interchange turbulence being present on the low-field side of the strike point and absent on the high-field side. This may directly relate to the phenomenology of strong interchange turbulence in the low-field side scrape-off layer outer mid-plane and the absence of turbulent transport in the high-field side scrape-off layer inner mid-plane [45].

13. I-modes can be operated within ~1 e-folding length of double-null with significantly reduced power fluxes to the inner divertor and increased levels of power sharing between the two outer divertors.

When combined with engineering specifications from the plasma magnetic equilibrium control, the fitted analytic formulas can provide guidance for scoping the divertor power handling requirements for future machines. However, scaling the fitted parameters to future machines is clearly an area of uncertainty that needs more work; it would benefit from both systematic experimental studies across multiple machines and also, if possible, an improvement in physics-based understanding of upstream profiles and transport of heat to the divertors. Integration with radiative divertor operation is another challenge, especially for the new class of ‘advanced’ divertors. Integration of double-null magnetic equilibria with radiative divertors was begun in DIII-D with flat-plate divertors [21]; however there is much more work to do. Changes in $\delta R_{\text{sep}}$ will undoubtedly change the power flow to each divertor. Maintaining both a balanced divertor power flow and also a stable radiation front in each divertor is a challenging integrated control problem that remains to be addressed.
Acknowledgements

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