Magnetic Reconnection Onset via Disruption of a Forming Current Sheet by the Tearing Instability

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The recent realization that Sweet-Parker current sheets are violently unstable to the secondary tearing (plasmoid) instability implies that such current sheets cannot occur in real systems. This suggests that, in order to understand the onset of magnetic reconnection, one needs to consider the growth of the tearing instability in a current layer as it is being formed. Such an analysis is performed here in the context of nonlinear resistive MHD for a generic time-dependent equilibrium representing a gradually forming current sheet. It is shown that two regimes, single-island and multi-island, are possible, depending on the rate of current sheet formation. A simple model is used to compute the criterion for transition between these two regimes, as well as the reconnection onset time and the current sheet parameters at that moment. For typical solar corona parameters this model yields results consistent with observations.

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Introduction. Magnetic reconnection is a fundamental process in plasmas, responsible, e.g., for solar flares, magnetospheric substorms, and sawtooth activity in magnetic fusion devices [1–3]. While reconnection itself has been intensely investigated, the problem of reconnection onset — i.e., the transition from a slow quiescent stage, when magnetic energy gradually accumulates, to an explosive energy release stage — is considerably less well understood and remains one of the most mysterious aspects of this fascinating phenomenon [4–9].

It is generally believed that reconnection requires quasi-two-dimensional regions of intense electric currents, the so-called current sheets (CSs), that sometimes form in a plasma through a variety of processes. Although several special cases of CS formation have been investigated in detail over the years [10–20], a solid, general understanding of CS formation is still lacking. Partly because of that, most numerical investigations of reconnection start with a fully developed thin current sheet as the initial condition, e.g., a Sweet-Parker reconnection layer [21, 22] in resistive MHD studies. However, the recent realization that long Sweet-Parker-like current sheets are super-Alfvénically unstable implies that, in reality, they can never form in the first place [23–29]. Similar considerations pertain to collisionless systems [30]. This underscores the importance of understanding reconnection onset in the context of a gradual current sheet formation process. Addressing this problem constitutes the main goal of this Letter.

Problem setup. We consider a current sheet whose defining parameters (the thickness $a$, the length $L$, and the reconnecting magnetic field $B_0$) are slowly evolving on some characteristic time scale $\tau_{dr}$, with the aspect ratio $L/a$ gradually increasing. With time, such a system becomes unstable to a spectrum of tearing modes, each characterized by wave-number $k(t)$, amplitude (magnetic island width) $w_N(t)$, and the number of islands $N \sim kL$. Our main goal is to compute the linear and nonlinear evolution of these modes in a forming current sheet, and thus to identify which mode is the first to exceed the width of the current sheet itself. The moment when this happens corresponds to the disruption of the forming current sheet by (one or many) primary magnetic islands, thus marking the transition between the slow energy build-up stage and the onset of fast energy release [31].

Our analysis is restricted to the resistive MHD description; however, the underlying conceptual framework remains the same if instead one wishes to consider weakly-collisional plasmas. A further assumption of this work is that the effects of background sheared flows (e.g., those associated with current sheet formation) on the tearing instability in both the linear [32] and nonlinear stages are ignored. This is justified as long as those flows are sub-Alfvénic, in which case we have found them to lead to only modest quantitative changes to our analysis, with no substantial effect on our conclusions [33].

Linear stage. Tearing modes are linearly unstable if the instability parameter $\Delta(k) > 0$ [34]. Let us consider the usual Harris-type magnetic equilibrium [35], for which $\Delta = 2(1/ka - ka)$ (all of the following results can be easily modified to conform to other equilibria). There are two possible linear regimes, distinguished by whether $\Delta \delta_m$ is small (‘FKR’ [34]) or order 1 (‘Coppi’ [36]). Here $\delta_m = [\gamma (kVA)^{-2}a^2\eta]^{1/4}$ is the width of the inner resistive layer, $\gamma$ is the instability growth rate, $V_A$ is the Alfvén velocity and $\eta$ is the
magnetic diffusivity. The growth rate in the FKR case is \( \gamma_{\text{FKR}} \sim \Delta^4/5 L^2/a^2 V_A^2 a^{-2/5} \eta^{1/5} \). The modes considered here have \( ka \ll 1 \); for these modes \( \Delta' a \sim 1/ka \) and hence \( \gamma_{\text{FKR}} \propto k^{-2/5} \). The fastest growing mode in the FKR regime is, therefore, the longest mode that fits in the current sheet, i.e., \( N \sim kL \sim 1 \), corresponding to

\[
\gamma_{\text{FKR}} \approx L^2/5 V_A^2 a^{-2/5} \eta^{1/5} = \tau_A^{-1} S_a^{-3/5} (L/a)^{3/5},
\]

where \( S_a \equiv aV_A/\eta \) (we assume \( S_a \gg 1 \)) and \( \tau_A \equiv a/V_A \).

In the Coppi case, we instead have \( \gamma_{\text{Coppi}} \approx k^2/3 V_A^2 a^{-2/3} \eta^{1/3} = \tau_A^{-1} S_a^{-1/2} \). The opposite scalings of \( \gamma \) with \( k \) in the FKR and Coppi regimes imply that the fastest mode is yielded by balancing the two expressions for \( \gamma \). We shall refer to this mode as the transitional or fastest Coppi mode; it is characterized by \( k_{\text{max}}^{\text{Coppi}} a \sim S_a^{-1/4} \), corresponding to \( \gamma_{\text{max}}^{\text{Coppi}} \sim \tau_A^{-1} S_a^{-1/2} \). Modes with \( k > \) \( k_{\text{max}}^{\text{Coppi}} \) in the FKR regime, while those with \( k < \) \( k_{\text{max}}^{\text{Coppi}} \) are in the Coppi regime; the existence of Coppi modes in a given current sheet, therefore, depends on its length \( L \), namely on whether the \( k_{\text{max}}^{\text{Coppi}} \) mode fits inside the layer, \( k_{\text{max}}^{\text{Coppi}} L > 1 \).

As progress proceeds, the increase in the sheet's aspect ratio gradually destabilizes higher and higher values of \( N \). Thus, there may be many modes independently undergoing linear tearing evolution; their amplitudes during the linear stage are much smaller than \( a \), so they do not yet affect current sheet formation or each other. For any given mode \( N \) we identify two important points in time, marking transitions between different stages in the mode's life: the time \( t_{\text{cr}}(N) \) marking the end of the linear stage, and the time \( t_{\text{tr}}(N) \) corresponding to the mode's transition from the FKR to the Coppi regime. The life path of the mode then depends on the relative ordering of \( t_{\text{cr}}(N) \) and \( t_{\text{tr}}(N) \). We define these two times as follows.

Firstly, both for the FKR modes (for any given \( N \)) and for the fastest growing Coppi mode \( N(t) = N_{\text{Coppi}} = k_{\text{max}}^{\text{Coppi}}(t) L(t) \), the linear tearing growth rate \( \gamma(N,t) \) increases with time as the current sheet develops (aspect ratio increases). For a given specific prescription for current sheet formation, one can then in principle compute the critical time \( t_{\text{cr}}(N) \) at which \( \gamma(N,t) \) overcomes the driving rate, i.e., \( \gamma(N,t_{\text{cr}}) \gtrsim \delta_{\text{dr}}^{-1} \). After the critical moment \( t_{\text{cr}}(N) \) is reached and until the end of the linear stage of the mode, its growth can be viewed as proceeding on a frozen background. Furthermore, neglecting logarithmic corrections, \( t_{\text{cr}}(N) \) essentially marks the end of the linear stage for the mode \( N \).

Secondly, we note that each mode \( N \) always starts out in the FKR regime, i.e., \( N_{\text{max}}^{\text{Coppi}}(t) < N \). However, as \( L/a \), and hence \( N_{\text{Coppi}} = (L/a) S_a^{-1/4} \), increase during the current sheet formation process, there may come a time \( t_{\text{tr}}(N) \) when \( N_{\text{max}}^{\text{Coppi}}(t) \) becomes equal to \( N \). At this point, provided that it is reached while the mode is still in the linear regime, i.e., \( t_{\text{tr}}(N) < t_{\text{cr}}(N) \), the mode transitions over to the Coppi regime. Importantly, higher \( N \) corresponds to higher \( t_{\text{tr}}(N) \).

Our main goal in the analysis of the linear tearing evolution is to find the mode that is the first to reach the end of its linear stage, i.e., the mode with the earliest \( t_{\text{cr}}(N) \). In general, which mode is the fastest-growing may change over time. At early times, the fastest growing mode is always the \( N = 1 \) (\( kL \sim 1 \)) FKR mode. If \( t_{\text{cr}}(1) \equiv t_{\text{cr}}(N = 1) < t_{\text{tr}}(1) \), then this mode remains in the FKR regime throughout its linear evolution. Furthermore, since \( t_{\text{tr}}(N) \) is a growing function of \( N \), then \( t_{\text{tr}}(N) > t_{\text{cr}}(1) \) and so all the other, higher-\( N \) modes also remain in the FKR regime throughout this time. Then, since at any given moment of time the \( N = 1 \) mode is the fastest-growing among all FKR modes, it reaches its \( t_{\text{tr}} \) first and thus "wins" the linear stage of the race. We will refer to this situation as the FKR scenario.

If, on the other hand, \( t_{\text{cr}}(1) > t_{\text{tr}}(1) \), i.e., if \( L/a \) exceeds \( S_a^{-1/4} \) before \( t_{\text{cr}}(1) \) is reached, then the \( N = 1 \) mode transitions over to the Coppi regime. After that moment, the fastest growing linear mode at any given time is the transitional mode \( N_{\text{Coppi}}(t) = (L/a) S_a^{-1/4} > 1 \). The mode that actually wins the linear stage in this case is the mode that reaches its \( t_{\text{cr}} \) essentially immediately upon transitioning to the Coppi regime (and hence becoming fastest growing) — i.e., before it can be overtaken by another Coppi mode. This mode is thus identified by the condition \( t_{\text{tr}}(N = 1) = t_{\text{cr}}(N) \).

At the end of the linear stage, the amplitude of any mode is \( w_{\text{N}}(t_{\text{cr}}) \sim \delta_{\text{in}}(k) \ll a(t_{\text{cr}}) \). In order to determine the mode that first disrupts the sheet, \( w_{\text{N}} \sim a \), and the moment when this occurs, we now need to consider the modes' nonlinear evolution.

**Nonlinear Stage.** The nonlinear evolution of a given mode \( N \) is governed by the product \( \Delta'(k_{\text{N}})w_{\text{N}} \). If it is small at the onset of the nonlinear phase, \( t = t_{\text{cr}}(N) \), then the mode enters the Rutherford stage [37] [38] characterized by algebraic growth, \( w_{\text{N}} \sim \eta^2 \Delta'(k_{\text{N}},t) \). This is indeed the case for modes that are in the FKR regime at the end of their linear evolution. Indeed, the condition for \( \delta_{\text{in}} \Delta' = (L/a) N S_a^{-2/5} \) to be small at \( t = t_{\text{cr}}(N) \) is equivalent to the condition, \( L(t_{\text{cr}})/a(t_{\text{cr}}) < N S_a^{1/4} \), for these modes to be in the FKR regime in the first place. Thus, these modes have a well-defined nonlinear Rutherford stage that continues as long as \( w_{\text{N}} \lesssim 1/\Delta' \).

However, as both the island width \( w_{\text{N}} \) and the current sheet aspect ratio grow, the product \( \Delta'(k_{\text{N}})w_{\text{N}} \sim (w_{\text{N}}/a)(L/Na) \) also grows. Eventually, at some \( t = t_{\text{X}}(N) \), the mode reaches the critical amplitude \( w_{\text{X,N}} \), such that \( \Delta'(k_{\text{N}})w_{\text{X,N}} \sim 1 \), for undergoing X-point collapse [15, 20, 39]. At this point, the Rutherford stage ends, the mode's inter-island X-points rapidly collapse to form thin secondary current sheets, and the growth of the mode greatly accelerates [20]. Since \( w_{\text{X,N}} \sim [\Delta'(k_{\text{N}})]^{-1} \sim k_{\text{N}} a^2 \ll a \) for modes with \( k_{\text{N}} a \ll 1 \) of interest to us here, we see that the dominant tearing mode
inevitably has to undergo X-point collapse before it can disrupt the sheet (which requires \( w_{N} \sim w_{\mathrm{disrupt},N} = a \)). On the other hand, one can show that the delay between these two events is rather short, since during the post-collapse phase the mode’s islands grow exponentially and very rapidly [20], and quickly reach the disruption size \( w_{\mathrm{disrupt},N} \sim a \).

This implies that the first mode to reach the X-point collapse condition will remain dominant throughout the subsequent evolution and will thus disrupt the current sheet. Our aim, therefore, is to identify this dominant mode and determine its collapse time, \( t_{X,N} \), which gives us a very good estimate for the time of the overall current-sheet disruption, \( t_{\mathrm{disrupt}} \).

First we consider the case \( t_{\mathrm{cr}}(1) < t_{\tau}(1) \), in which the first mode to reach the end of its linear stage is the FKR mode \( N = 1 \). Then, by comparing the \( N \)-dependencies of \( t_{\mathrm{cr}}(N) \) and \( t_{\tau}(N) \), one can show that \( t_{\mathrm{cr}}(N) < t_{\tau}(N) \) for all the other modes. This means that there are simply no Coppi modes in this case — all modes remain in the FKR regime throughout their linear evolution and then individually transition to the Rutherford stage.

Furthermore, as can be seen immediately from the Rutherford growth equation, \( w_{N} \approx \eta \Delta'(k_{N}) \sim k^{-1} \sim N^{-1} \), and so lower-\( N \) modes grow faster than higher-\( N \) modes during the Rutherford stage. This implies that the \( N = 1 \) mode will reach X-point collapse first. More rigorously, integration of \( \dot{w}_{N} \approx \eta \Delta'(k_{N}) \) yields

\[
w_{N}(t) = w_{N}(t_{\mathrm{cr}}) + \frac{2\eta}{N} \int_{t_{\mathrm{cr}}}^{t} \frac{L(t')}{a^{2}(t')} \, dt',
\]

where \( w_{N}(t_{\mathrm{cr}}) \approx \delta_{\mathrm{in}} \ll a(t_{\mathrm{cr}}) \).

For simplicity, let us consider the case when \( a \) is a decreasing function of time \( (L \text{ and } B_{0} \text{ may also be changing}), da/dt < 0 \); we can then parameterize the current sheet evolution by \( a \) instead of \( b \). Then, Eq. (1) yields

\[
w_{N}(a) \approx \frac{\eta}{N} \int_{a[t_{\mathrm{cr}}(N)]}^{a} \frac{L(a)}{a^{2}} t'(a) \, da,
\]

where \( t'(a) \equiv dt(a)/da < 0 \) and where we have ignored the initial plasmoid width \( w_{N}(t_{\mathrm{cr}}) \) and factors of 2. We then see that \( a_{X,N} \), the value of \( a \) at which \( w_{N}(a) \Delta' \sim 1 \), is given implicitly by

\[
N = \eta \frac{L(a_{X,N})}{a_{X,N}} \int_{a[t_{\mathrm{cr}}(N)]}^{a_{X,N}} \frac{L(a)}{a^{2}} t'(a) \, da.
\]

Since the nonlinear Rutherford stage is expected to last much longer than the linear one (given that the Rutherford growth is algebraic in time, whereas linear-stage growth is exponential), we can neglect the lower bound in this integral. Then, since \( t'(a) < 0 \), we see that the integral on the right-hand side (RHS), and hence the entire RHS, decrease with \( a_{X,N} \). This means that smaller-\( N \) Rutherford modes correspond to larger \( a_{X,N} \) and thus reach the X-point collapse faster.

It is easy to show that the same conclusion is reached if one considers current sheet formation associated not with the thinning of the layer \( (a < 0) \) but instead with its stretching and lengthening, \( L > 0 \), at a fixed thickness \( a(t) = a_{0} \); this case corresponds to the channel flows due to the magnetorotational instability [40, 41] in accretion disks.

To sum up, the nonlinear Rutherford evolution of FKR-type modes does not change the result of the linear analysis: if the most unstable linear mode was in the FKR regime \( (N = 1 \text{ FKR mode}) \), then this mode will continue to dominate during the nonlinear Rutherford stage and will be the first to reach the X-point collapse. Ultimately, this is the mode that disrupts the sheet.

Now we consider the nonlinear evolution for the second case, \( t_{\mathrm{cr}}(1) > t_{\tau}(1) \), in which the first mode to reach the end of the linear stage is the fastest Coppi mode \( N = N_{\text{Coppi}}^\text{max} \). One can easily show that \( \delta_{\mathrm{in}} \Delta' \sim 1 \) at \( t = t_{\mathrm{cr}} \) for this mode. This means that the Rutherford regime is essentially absent and hence X-point collapse occurs promptly, soon upon the mode’s arrival at the nonlinear stage. Correspondingly, this mode is the first to undergo X-point collapse and to ultimately disrupt the current sheet; the disruption time is then comparable to the time it has spent in the linear regime, i.e., \( t_{\mathrm{disrupt}} \sim t_{\mathrm{cr}}(N_{\text{Coppi}}^\text{max}) \).

The main general conclusion we draw from these considerations is that the outcome of the nonlinear evolution of both FKR/Rutherford and Coppi tearing modes in a gradually forming current sheet is the same as that of the linear stage. Namely, the first mode to reach the end of its linear stage (by meeting the condition \( \gamma \tau_{\mathrm{dr}} = 1 \)) will also proceed to be the first to undergo the X-point collapse and subsequently will disrupt the current sheet.

The above formalism is general and can be applied to any process of current sheet formation if the functional forms for the time evolution of the sheet parameters are known. A simple but general example is analyzed next.

**Example: Chapman-Kendall-like current sheet formation.** Consider an X-point configuration given by \( \phi = v_{\mathrm{dr}} xy/L(t), \psi = B_{0}/2[x^{2}/a(t) - y^{2}/L(t)] \), where \( \phi \) is the (incompressible) flow stream function, \( \psi \) is the magnetic flux, \( B_{0} \equiv \text{const} \) and \( v_{\mathrm{dr}} = \text{const} \) represents the plasma velocity driving the current sheet formation (cf. [10]; [42]). Substituting these expressions into the ideal reduced-MHD equations [43], one obtains

\[
a(t) = a_{0}L_{0}/(L_{0} + 2v_{\mathrm{dr}} t), \quad L(t) = L_{0} + 2v_{\mathrm{dr}} t,
\]

where \( a_{0} \equiv a(t = 0) \) and \( L_{0} \equiv L(t = 0) \). The current sheet formation driving timescale \( \tau_{\mathrm{dr}} = L_{0}/v_{\mathrm{dr}} \) then becomes \( \tau_{\mathrm{dr}} \sim t \) for \( t \gg L_{0}/v_{\mathrm{dr}} \).

It is convenient to introduce two main dimensionless parameters of the system: the Alfvén Mach number (which quantifies the ideal-MHD current-sheet formation drive), \( M_{\mathrm{dr}} \equiv v_{\mathrm{dr}}/V_{A} \), assumed to be \( \lesssim 1 \), and the initial Lundquist number \( S_{0} \equiv (a_{0}L_{0})^{1/2}V_{A}/\eta \gg 1 \).
Focusing on late times, $2\tau_{dr} t \gg L_0$ (and hence $L \gg a$), we see that the tearing instability parameter is $\Delta(t) = (16/N) [v^3_t/(a^2 L_0^2)] t^3$. The transition from the FKR to the Coppi regime for the $N = 1$ mode occurs when $(L/a) S_{a}^{-1/4} \sim 1$, i.e., $\tau_{tr}(1)/\tau_{A0} \sim M_{dr}^{-1} S_0^{1/9}$, where $\tau_{A0} \equiv (a_0 L_0)^{1/2}/V_A$. The critical time for the $N = 1$ FKR mode, $\gamma_{FKR}^\text{max} L(\tau_{cr})/v_{dr} \sim 1$, is $t_{FKR}^\text{cr}(1)/\tau_{A0} \sim M_{dr}^{-12/17} S_0^{1/17}$ [44]. The condition that the fastest growing linear ($N = 1$) mode is in the FKR regime at this time is simply $t_{FKR}^\text{cr}(1) < t_{tr}(1)$; this yields a condition on the drive:

$$M_{dr} < M_{dr,c} \equiv S_0^{-2/9}. \quad (5)$$

If this condition is not satisfied, then the $N = 1$ mode and several higher-$N$ modes transition to the Coppi regime while still in the linear stage. The mode number corresponding to the fastest growing mode then changes (grows) with time as the current sheet parameters evolve, according to $N_{\text{Coppi}}^\text{max} \sim (L/a) S_{a}^{-1/4}$. As discussed above, the mode that reaches the end of its linear evolution first and thus wins the linear stage is the one that undergoes the transition from FKR to the Coppi regime just before that moment, i.e., the mode with $t_{tr} \approx t_{cr}$. For the current sheet formation model (4) discussed here, this dominant Coppi mode is

$$N_{\text{Coppi}}^\text{max} \sim \left[ L/a S_{a}^{-1/4} \right] t_{tr}^{C_{tr}} \sim M_{dr}^{9/10} S_0^{1/5} \quad (6)$$

and its critical time and current sheet dimensions are

$$t_{cr}^{\text{Coppi}} \approx t_{tr}(N_{\text{Coppi}}^\text{max}) \sim M_{dr}^{-3/5} S_0^{1/5} \tau_{A0}, \quad (7)$$

$$a_{cr}^{\text{Coppi}}/(a_0 L_0)^{1/2} \sim M_{dr}^{-2/5} S_0^{-1/5}, \quad (8)$$

$$L_{cr}^{\text{Coppi}}/(a_0 L_0)^{1/2} \sim M_{dr}^{-1/5} S_0^{1/5}. \quad (9)$$

and hence $(L/a)^{\text{Coppi}}_c \equiv (L/a) t_c^{\text{Coppi}} \sim M_{dr}^{4/5} S_0^{2/5}$. As a consistency check, we see that $t_{tr}(1) < t_{cr}^{\text{Coppi}}$ if $M_{dr} \gg S_0^{-2/9}$, which is required for the dominant mode to be in the Coppi regime by the end of the linear stage in the first place. Also, it is instructive to note that $(L/a)^{\text{Coppi}}_c \sim [S_L (t_c^{\text{Coppi}})]^{1/3} M_{dr}^{2/3}$ (where $S_L (t) \equiv L(t)V_A/\eta$), which generalizes the scaling obtained in Ref. [28] for $M_{dr} = 1$.

In the nonlinear phase, if the condition (5) is satisfied, the $N = 1$ mode continues to dominate and undergoes Rutherford evolution described by $\dot{\omega}_1 \sim \eta \Delta'(N = 1)$, yielding $\omega_1(t) \approx \omega_1(\text{FKR}) + 4\eta v_{dr}^3 t/t_{cr}^{\text{FKR}} (t^4 - t_{cr}^{\text{FKR}})^{1/2} / (a_0^2 L_0^2)$, where $\omega_1(\text{FKR}) \approx \dot{\omega}_1$ is the island width at the beginning of the Rutherford stage. As discussed above, the Rutherford stage continues until $X$-point collapse. It can be checked a posteriori that, if we are in the regime where the FKR mode dominates the linear evolution, i.e., if the condition (5) is satisfied, then the Rutherford stage lasts much longer than $t_{cr}^{\text{FKR}}(1)$. One can then also show that the critical island width triggering $X$-point collapse, $w_{X,1}$, is greater than $w_1(t_{cr}^{\text{FKR}})$. Therefore, the growth of the $N = 1$ FKR mode throughout most of the Rutherford stage is described by (ignoring factors of order unity):

$$w_1(t) \sim \eta v_{dr}^3 a_0^2 L_0^2 t^4. \quad (10)$$

Then, the time for this mode to reach the collapse condition $w_1(t_{cr}) \sim 1 \Rightarrow w_1 = w_{X,1} = \Delta'(t_{X,1})^{-1}$ is

$$t_{X,1} \sim \tau_{A0} M_{dr}^{-6/7} S_0^{1/7} \gg t_{cr}^{\text{FKR}}, \quad (11)$$

the corresponding island width is $w_{X,1} \sim (a_0 L_0)^{1/2} S_0^{3/7} M_{dr}^{-3/7}$, and the current sheet parameters at this time are

$$a_{X,1}/(a_0 L_0)^{1/2} \sim M_{dr}^{-1/7} S_0^{-1/7}, \quad (12)$$

$$L_{X,1}/(a_0 L_0)^{1/2} \sim M_{dr}^{-1/7} S_0^{1/7}, \quad (13)$$

$$[L/a]_{X,1} \sim M_{dr}^{2/7} S_0^{7/4}. \quad (14)$$

As we have argued, because of the rapid acceleration of the mode’s growth following the collapse [20], $t_{\text{disrupt}} \sim t_{X,1}$, and thus these expressions yield fairly good practical estimates for the final parameters at the moment of disruption $w = a$.

If, on the other hand, Eq. (5) is not satisfied, then both the linear and the nonlinear evolution are dominated by the Coppi mode with $N_{\text{Coppi}}^\text{max}$ given by Eq. (6). This mode undergoes $X$-point collapse and quickly leads to current sheet disruption essentially as soon as it becomes nonlinear, i.e., $t_{\text{disrupt}} \sim t_{X,1}^{\text{Coppi}} \sim t_{cr}^{\text{Coppi}}$. Consequently, the current sheet thickness, $d_X^{\text{Coppi}}$, and length, $L_X^{\text{Coppi}}$, at the moment of disruption are well approximated by the corresponding values at $t_{cr}^{\text{Coppi}}$, i.e., given by Eqs. (8–9).

It is worth noting that both Eqs. (12–14) and Eqs. (8–9) scale only very weakly with the two key input parameters $M_{dr}$ and $S_0$, pointing to a certain universality of the FKR/Rutherford and the Coppi evolution scenarios: indeed, in each of these regimes one will find reasonably similar estimates for a wide range of $M_{dr}$ and $S_0$.

Also notice that in both the FKR and Coppi cases $a_X$ is much larger than the corresponding Sweet–Parker current sheet thickness $\delta_{SP} \sim L_X S_X^{-1/2}$, where $S_X = L_X V_A/\eta$ is the Lundquist number at the time of $X$-point collapse. The ensuing disruption of the current sheet implies, therefore, that a global-scale Sweet–Parker layer is never formed, as we anticipated.

As an application, let us consider approximate parameters for a typical solar flare: $a_0 = L_0 = 10^4$ km, $n_c = 10^{10}$ cm$^{-3}$, $B_0 = 100$ G, resulting in $V_A \sim 2000$ km/s, $\tau_{A0} \approx 5$ s, and $S_0 \approx 3 \times 10^{13}$. Eq. (5) yields roughly $M_{dr,c} \approx 0.001$, corresponding to $v_{dr,c} \approx 2$ km/s, comparable to typical photospheric velocities. It is likely that in the real corona a broad range of drives are available. As such, let us contemplate both the FKR and Coppi cases by considering $M_{dr} = M_{dr,c} = 10^{-3}$ (FKR, $N \sim 1$) and $M_{dr} = 0.05$ (Coppi; this case corresponds
to $v_{dr} \approx 100$ km/s, as may arise, e.g., due to ideal-MHD instabilities or loss of equilibrium in the corona driving the emergence of coronal mass ejections). We obtain: (i) $M_{dr} = 10^{-3} \rightarrow a_{\text{FKR}}^{\text{disrupt}} \approx 300$ km, $L_{\text{disrupt}} \approx 3 \times 10^5$ km, $t_{\text{FKR}}^{\text{disrupt}} \approx 40$ hours; (ii) $M_{dr} = 0.05 \rightarrow a_{\text{Coppi}}^{\text{disrupt}} \approx 70$ km, $L_{\text{Coppi}}^{\text{disrupt}} \approx 1.5 \times 10^6$ km, $t_{\text{Coppi}}^{\text{disrupt}} \approx 4$ hours, and $N \approx 30$. These are reasonable numbers (see, e.g., [9, 45–47]), especially in light of the crudeness of the current sheet formation model that we are considering here. In particular, both $t_{\text{FKR}}^{\text{disrupt}}$ and $t_{\text{Coppi}}^{\text{disrupt}}$ are consistent with observed pre-flare energy-buildup times. In addition, note that the smallest meaningful length scale in our problem, $\delta_m$, remains much larger than the ion kinetic scales (the skin depth, $c/\omega_{pi} \approx 2$ m and the Larmor radius, $\rho_i \approx 0.1$ m) in all cases: $\delta_m(t_{\text{ext}}) \approx 100$ – 300 m. This validates our usage of the resistive MHD plasma description for reconnection onset in the solar corona in this particular example.

Conclusions. In this study we have developed a general conceptual framework connecting two important and related phenomena that have hitherto been considered separately: large-scale ideal MHD processes leading to thin current sheet formation and magnetic energy accumulation, and the onset of fast energy release through reconnection. In our picture, the immediate outcome of this sequence of events is the disruption (and thus replacement) of the forming current sheet by a chain of primary magnetic islands generated by the tearing instability. Our study is substantially different from, and more fundamental than, previous related work on the tearing instability of reconnecting current sheets [28, 32], which has focused exclusively on the linear evolution of a time-independent current sheet and has not considered if and how the FKR regime transitions into the Coppi regime. In contrast, we have considered a time-evolving current sheet at an arbitrary current-sheet formation rate, computed the pertinent timescales related to various unstable tearing modes, and analyzed the order in which these various processes happen during both the linear and nonlinear evolution. Our analysis has allowed us to predict for the first time the moment at which the current sheet is disrupted (the reconnection onset), the number of primary magnetic islands that disrupt the sheet, and the final current sheet properties at the time of disruption, and elucidate their dependence on the Lundquist number and the current sheet formation drive. In particular, our analysis reveals that two distinct regimes are possible: the FKR/Rutherford regime, in which the current sheet is disrupted by only one or two islands; and the Coppi regime, where, instead, the sheet is disrupted by a large number of islands. Both scenarios can be relevant to experimental, astrophysical and space systems, including solar flares, where they yield reasonable estimates for flare onset time [9].

Although we have restricted ourselves to the MHD description, the conceptual framework outlined in this Letter is completely general and can be extended to collisionless plasmas provided that the linear and nonlinear regimes of the tearing instability are understood in the particular collisionless formulation that one chooses to adopt; this is indeed necessary in order to address the onset problem in two other prominent contexts: the sawtooth instability in tokamaks [48], and reconnection in the Earth’s magnetotail [5].

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Note that the dominant mode discussed here characterizes only the onset of reconnection; during the subsequent reconnection stage proper, a power-law distribution of island sizes will likely form [49–52].

As a side remark, the current sheet’s aspect ratio at this time, $[L/a]_{FKR}^{cr} \sim \frac{M}{17} \frac{S_{a}^{14/17}}{17}$, can be expressed in terms of $S_{a}(t_{FKR}^{cr}) \sim \frac{M}{17} \frac{S_{a}^{14/17}}{17}$ as $[L/a]_{FKR}^{cr} \sim M^{5/17} [S_{a}(t_{FKR}^{cr})]^{3/7}$, which generalizes Bulanov’s [32] Alfvénic-drive ($M_{dr} = 1$) result $L/a \sim S_{a}^{3/7}$ to the case of arbitrary drive $M_{dr}$ and shows the mutual consistency of ours and theirs approaches.

Rutherford [37] assumes that only the lowest harmonic, $N = 1$, is tearing unstable, whereas all others are strongly stable. In fact, it has been demonstrated [53] that Rutherford’s results remain valid even if $N > 1$ modes are also unstable, as we are implicitly assuming here.