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Design of a Metamaterial Based Backward-Wave Oscillator

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Abstract—In this paper we present the design of a microwave generator using metamaterials (MTMs) in a negative index waveguide interacting with a high power electron beam. The microwave structure is constructed by inserting two MTM plates made of complementary split-ring-resonators (CSRRs) into a rectangular waveguide. Electromagnetic simulations using the HFSS code confirm the presence of a negative index TM-like mode suitable for use in a backward-wave oscillator. Particle in cell (PIC) simulations using the CST Particle Studio code are performed to evaluate the efficiency of the metamaterial BWO (MTMBWO) and the performance of the device in use with a 500 keV, 80 A electron beam. Efficiency of >14% is observed with saturated output at 5.75 MW. The MTMBWO requires about 250 ns to reach saturation of the output power. The MTMBWO is also modeled by representing the MTM plates, which consist of CSRRs, as dielectric slabs whose effective permittivity is given by a Lorentzian model. The dielectric slab model is also simulated with the CST PIC code and shows good qualitative agreement with the more exact model using the actual CSRRs. A structure was fabricated from brass to test the theoretical predictions of the microwave transmission vs. frequency of the negative index waveguide. Test results at low power testing using a vector network analyzer showed very good agreement with simulation for the excitation of the negative index TM-like mode near 2.6 GHz. The proposed structure appears to be promising for use in a MTMBWO high power microwave generator.

Index Terms—Metamaterial, BWO, Vacuum Electronics, Plasma Waves

I. INTRODUCTION

METAMATERIALS (MTMs) have unique electromagnetic properties with the potential to open new possibilities in the design of high power microwave devices. One unique property of MTMs is the ability to support negative index modes due to a simultaneous negative effective permeability and permittivity [1], [2]. Sub-wavelength resonant structures are used to create macroscopic negative effective parameters. There has been much experimental and theoretical work to investigate the application of these so called double-negative metamaterials in various engineering applications, including perfect lens design, electromagnetic cloaking, advanced antenna design, accelerator applications, and coherent microwave generation [3], [4]. Of interest to this paper is the use of MTMs in microwave generation, in which a limited amount of theoretical and experimental work has already been performed [5], [6], [7], [8], [9]. The challenges faced in the construction of most vacuum electron devices (high power microwave fields, vacuum environment, frequency tunability, etc.) place unique design considerations on a MTM which are not present in optics or low power microwave electronics.

The use of high power electron beams to produce or amplify coherent microwave radiation is a well established yet active area of research [10]. Traveling wave tubes (TWTs) and backward wave oscillators (BWOs) are both common types of microwave generators that rely on Cerenkov or Smith-Purcell radiation from an electron beam interacting with a slow wave (phase velocity $v_{ph} < c$) to produce coherent radiation. While the physical description of both devices is quite similar, the TWT amplifies a microwave signal traveling with the electron beam and the BWO generates a backward wave (group velocity $v_{gr} < 0$) traveling in the opposite direction of the beam.

Arrays of split-ring resonators (SRRs) were introduced as a means of achieving an effective negative permeability in a bulk material [11]. When used in conjunction with an array of metallic posts, one can create a negative index medium, having simultaneous negative permeability and permittivity. The electric analog of the SRR, the complementary-SRR (CSRR) has been shown to produce a negative permittivity [12]. In addition, a TM mode in a below-cutoff waveguide is equivalent to a negative effective permeability medium. In order to create a waveguide that supports a negative index mode, we propose the use of CSRRs in a below-cutoff waveguide. Since a TM mode is of interest for interacting axially with an electron beam, two parallel CSRR plates running along the electron beam trajectory are used. The geometrical arrangement of the MTM layers is shown to affect the properties of the modes they support, which has also been discussed in Ref. [13].

In a mode with negative dispersion, power flows anti-parallel to the electron beam’s motion. While the interaction is similar in nature to a traditional BWO, it is unique in that a conventional BWO relies upon the interaction of an electron beam with spatial harmonics of a slow wave structure. In a negative index guide, a mode with negative group velocity is supported by the simultaneous negative permittivity and permeability of the guide. A related approach in creating structures that support modes with negative group velocity makes use of plasmonic waveguides [14], [15], [16], [17], [18], [19]. The use of MTMs in a waveguide provides a means to artificially duplicate the dispersive effect of the plasma without creating the problem of actually producing and sustaining a plasma in a microwave tube.

In addition to the novel physical motivation for their study,
MTM based vacuum electron devices may have several key advantages over conventional devices from a purely practical and engineering standpoint. MTMs are frequency selective, and thus will be useful for applications that require precise frequency control. It is also possible that MTM based vacuum electron devices could be easier to manufacture than conventional tubes. This paper examines the use of a completely planar MTM structure created from a metal plate.

II. STRUCTURE DESIGN AND ELECTROMAGNETIC SIMULATIONS

In Fig. 1 we show a specific design for a proposed MTM based BWO (MTMBWO) structure that will operate in S-Band (2-4 GHz). Other frequency devices can also be designed by scaling the structure shown. The MTMBWO is constructed by placing two periodic MTM plates into a rectangular waveguide. Schematics of one period of the MTM plate and of the structure are shown in Fig. 1a and Fig. 1b, respectively. The resonant frequency of the MTM plates is designed to be below cutoff for TM modes in the waveguide. The MTM plates are created by machining CSRRs with period \( p = 8 \) mm and width \( h = 41 \) mm along two metal plates of thickness \( t = 1 \) mm. The resonators have a slot width of \( a = 2.5 \) mm. These plates are then placed with a separation of \( d = 42 \) mm in a standard size WR340 waveguide (inner dimensions 86 mm \( \times \) 43 mm). In Fig. 1c, a structure is shown with output coupling included, which is WR284 (inner dimensions 76 mm \( \times \) 34 mm) waveguide mated to the MTMBWO structure. The overall length of the structure is 440 mm. The cutoff frequency, \( f_c \), of the lowest order TM mode in a rectangular waveguide of transverse dimensions \( l_1 \times l_2 \) is given by the relation

\[
f_c = \frac{1}{2\pi\sqrt{\epsilon_0\mu_0}} \sqrt{\frac{\pi^2}{l_1^2} + \frac{\pi^2}{l_2^2}}
\]

where \( \epsilon_0 \) and \( \mu_0 \) are the free space permittivity and permeability, respectively. For an empty 43 mm \( \times \) 86 mm rectangular waveguide \( f_c = 3.90 \) GHz.

The eigenmode solver of the High Frequency Structure Simulator (HFSS) code is used to simulate the eigenmodes of one period of the total structure. The eigenmode simulations also generate 3D \( E \) and \( H \) field vectors in the structure which can be used to estimate BWO performance parameters (coupling impedance, start current, etc) [20], [21]. Because of symmetry along the center of the structure in the plane parallel to the CSRR plates, a perfect-H boundary was used to reduce simulation time. In Fig. 2 the dispersion relation calculated from the HFSS simulation is shown, along with both the beam line \( 2\pi f = k_z v_0 \) and the light line \( 2\pi f = k_z c \). Here \( 2\pi f \) is the angular frequency, \( c \) is the speed of light, \( k_z \) is the wavenumber, and \( v_0 = 0.86 \) \( c \) is the velocity of a 500 keV electron. The structure supports two negative index modes, one with \( E_z \approx 0 \) at the axis at \( \sim 2 \) GHz and a TM-like \( (H_z = 0 \) at the axis) at \( \sim 2.5 \) GHz. Because the axial field is small, the negative index mode at 2 GHz is not expected to interact strongly with the electron beam and is also not observed in PIC simulation. The beam line intersects the dispersion relation for the negative index TM-like mode at approximately 2.65 GHz, where the group velocity \( v_{gr} \equiv \frac{\partial \omega}{\partial k} < 0 \) and the phase velocity \( v_{ph} \equiv \frac{c}{k} > 0 \). The magnitude of the group velocity of the negative index TM-like mode at the point of intersection with the beam line is \( |v_{gr}| \approx 0.075c \).

Of particular interest is the performance of the MTMBWO when compared against a more conventional BWO such as a rippled wall device. In such a microwave generator there is a start current for oscillation, \( I_{st} \), above which the device will oscillate with zero input signal. The start current depends on the device geometry, mode of interaction, and beam parameters. Traditionally one operates the BWO at \( \sim 3 \times \) the start current. Increasing the current well beyond the start current \( (\times \sim 7) \) can result in auto-modulation of the output power as the device enters a stochastic regime. This type of behavior has been observed in both theory and experiment as shown, for example, in Ref. [22]. In order to estimate the start current for the MTMBWO, we use a loss-less linear theory outlined...
in Ref. [21]. The coupling impedance is given by the relation

$$Z = \frac{|E_w|^2}{2k \omega P}$$  \hspace{1cm} (2)

where $|E_w|$ is the component of the electric field parallel to the direction of and in phase with the electron beam, $P$ is the power flux, and $k_{z0}$ is the wavenumber. The numerical value of the coupling impedance is calculated from the electric and magnetic field profiles generated by the HFSS eigenmode simulation for the negative index TM-like mode and is found to be $Z = 46 \, \Omega$. The start current is then readily calculated from the equation

$$I_{st} = 4U_0 \frac{(CN)_{st}^3 \lambda_2^5}{ZL^3}$$  \hspace{1cm} (3)

where $U_0$ is the beam energy, $\lambda_2 = \frac{2 \pi}{k_2}$ is the longitudinal wavelength (9.6 cm), $L$ is the total length of the structure, $N = L/\lambda_2$ is the number of longitudinal wavelengths, and $C$ is the Pierce parameter given by the equation

$$C^3 = \frac{I_0 Z}{4U_0}$$  \hspace{1cm} (4)

Here $I_0$ is the beam current. From Table 8.1 in Ref. [21] the start condition $(CN)_{st}$ is taken to be 0.314 for zeroth order axial harmonic operation. From Eq. 3 the start current is plotted in Fig. 3 as a function of structure length.

III. PIC SIMULATION RESULTS AND DISCUSSION

We use the PIC solver of CST Particle Studio to investigate the performance of the MTMBWO utilizing a relativistic electron beam. A variable current (60, 80, and 100 A) and a 500 keV ($v_0 = 0.86 \, c$) beam of radius 2.2 mm is used for the simulations. The electron beam simulated was a DC beam with a 4 ns rise time to reach full current from zero initial starting current. A uniform and axial magnetic field of 1.5 kG is used for all simulations. Output ports are shown in Fig. 1c and were used to record the power generated and coupled out of the structure.

Fig. 4 shows the output power as a function of time for a simulation of a 438 mm long MTMBWO structure using an 80 A electron beam. The device takes 260 ns to reach a stationary output power regime where 5.75 MW average power was produced at 2.595 GHz. Shown in Fig. 5 is a plot of the electron density for the same simulation taken at 400 ns which clearly demonstrates the formation of electron bunches in the device. As the bunches travel along the axis, they start to break up as they lose energy to the wave and due to space charge.

A plot of the electron trajectories is shown in Fig. 6a. This plot is also from the same simulation and taken at 400 ns. The energy of individual particles is indicated by their color, and distinct electron bunches are visible. The beam size is affected by the RF field and a slight increase in the beam radius is visible as the beam exits the MTMBWO structure. The electric field magnitude at this time is shown in Fig. 6b. The field grows from zero at the electron beam exit to its peak value at the beam entrance, indicating the backw ardness of the mode. The field strength is greatest close to the MTM plates.

In Fig. 7 the output power and spectrum are shown for a 60, 80, and 100 A beam for a fixed length of 462 mm. Using
Fig. 6. a) Cross section of the MTMBWO showing particle trajectories for a 438 mm long structure. The individual energy of each particle is indicated by the color scale. b) Electric field magnitude in the MTMBWO. Both snapshots are taken 400 ns after the injection of the electron beam, i.e. the device has reached saturation.

Fig. 7. Output powers and power spectra of stationary outputs for currents of 60, 80, and 100 A. The structure length for all three simulations was 462 mm.

the results of Eq. 3 and Fig. 3, the nominal start current for the MTMBWO is 12 A for a 462 mm long structure. A clear change in the behavior of the MTMBWO is visible as the current is increased from 80 to 100 A, corresponding to 6.7 and 8.3 times the start current respectively. At 100 A there is auto-modulation of the output power, which is indicated by the multi-frequency operation in the output spectra. Note that for an interaction length of 438 mm and a beam current of 80 A the auto-modulation instability is absent, which is shown in Fig. 4.

Output efficiency is an important figure of merit when comparing the MTMBWO against more traditional BWO designs. The output efficiency, calculated by taking the average steady state power output at the output ports divided by the beam power is shown in Fig. 8 as a function of structure length for three different beam currents. A peak efficiency of 14.5 % is predicted for the 438 mm long structure using 80 A.

IV. EFFECTIVE MEDIUM PIC SIMULATION RESULTS AND DISCUSSION

A standard and convenient way to interpret the dispersive qualities of a MTM is by use of an effective medium model, which treats the collective electromagnetic behavior of individual MTM resonators as a bulk material with an effective permittivity or permeability [23]. Comparison between such a model and actual MTM resonators can verify that the resonators are indeed acting as a MTM (i.e. providing an effective negative permittivity or permeability at the frequency of interest). More practically, the simulation of a MTM using a PIC code can be computationally intensive, as accurately meshing the sub-wavelength resonators can make for extremely large mesh sizes. Thus substituting the actual MTM resonators with a bulk material can save simulation time. A similar approach to simulate a MTM interacting with an electron beam using an effective medium was used in Ref. [5].

We again use the PIC solver in CST Particle Studio to simulate a MTM based BWO similar to the MTMBWO investigated in the previous section. Instead of using CSRR based MTM plates, an isotropic effective medium is inserted into the waveguide which models the dispersive qualities of the CSRRs. The model is shown in Fig. 9, where the same outer waveguide as the MTMBWO (WR340) is used to provide a negative permeability. Using the Drude model, the permeability of a waveguide follows the expression [3], [24]

$$\mu_{eff} = 1 - \frac{\omega_c^2}{\omega^2}$$

where \(\omega_c = 2\pi \cdot 3.9 G Hz\) is the cutoff frequency of the fundamental TM mode, and the numerical value was calculated for the WR340 waveguide using Eq. 1. For \(\omega < \omega_c\), the permeability of the waveguide is negative.

Coupling power out of the structure is identical in both the effective medium model and MTMBWO. Two dielectric slabs
of width \( w = 12 \text{ mm} \) are placed in the same location as the CSRR loaded plates in the MTMBWO. The overall length of the structure is \( L = 438 \text{ mm} \) long. The electron beam parameters used, \( I = 80 \text{ A}, U_0 = 500 \text{ keV} \) and beam radius \( r_b = 2.2 \text{ mm} \), are consistent with those used in the simulations of the MTMBWO. A Lorentz model for the dispersion of the effective medium is used, which is given by the relation \([3], [24]\)

\[
\epsilon_{\text{eff}} = 1 - \frac{\omega_p^2}{\omega^2 - \omega_0^2}
\]

where \( \omega_p \) is the plasma frequency and \( \omega_0 \) is the resonant frequency of the CSRRs. The particular numerical values are chosen for these two frequencies from the dispersion relation obtained by the HFSS eigenmode solver in order to mimic the dispersion of the MTMBWO. Since the permittivity is given by \( \epsilon = \frac{\omega_p^2}{\omega^2} \), the resonance and plasma frequencies in Eq. 6 are determined from the dispersion relation (shown in Fig. 2) by letting \( \epsilon \to \infty \) and \( \epsilon \to 0 \), which corresponds to letting \( k_z p \to \infty \) and \( k_z p \to 0 \), respectively. We find the resonance to be at \( \omega_0 = 2\pi \cdot 2.3 \text{ GHz} \) and the plasma frequency is determined to be \( \omega_p = 2\pi \cdot 1.7 \text{ GHz} \).

Fig. 10a shows the particle trajectories and formation of electron bunches due to the BWO interaction. The magnitude of the electric field is displayed in Fig. 10b. Comparing this with the fields shown for the CSRR based MTMBWO in Fig. 6b we see a similar field structure in the region of the guide outside of the dielectric slab (i.e. in the beam tunnel and above and below the slab). The output power as a function of time and the Fourier transform of this signal is given in Fig. 11. The device saturates at 3 MW and the frequency is 2.62 GHz, which is near the output frequency of the MTMBWO of the same length at 2.595 GHz and approximately half of the saturated power. Different slab widths \( w \) were investigated, with 3 mm to 20 mm having a similar frequency response with some variation in output power and saturation time. For \( w = 1 \text{ mm} \) the slab was too thin for the beam to couple to the guide, and as the slab was made very large (> 20 mm) the presence of the slab interfered with power coupling out of the guide. As the slab thickness has been shown to be important in the overall response of MTM layers, a more detailed analysis would be necessary to determine exact slab parameters to replicate the CSRR loaded plates [25]. Although the results of the analysis provided by this model are not in exact agreement with the exact MTMBWO structure, they do provide valuable physical insight. \([26]\)

V. STRUCTURE COLD TEST RESULTS

To validate the electromagnetic response of the MTMBWO design, measurements of microwave transmission in a test MTMBWO structure made of brass were performed using a vector network analyzer (VNA). Experimental measurements of various coupling schemes to MTM devices has been investigated due to the importance for real-world MTMs [27], [28]. The test structure and schematics are shown in Fig. 12. The structure is identical to that shown in Fig. 1 except with \( p = 7 \text{ mm}, d = 16 \text{ mm}, \) and \( a = 2 \text{ mm} \). The overall structure length was 20 periods (160 mm). The MTM plates were aligned in
the WR340 waveguide by end supporting plates, and the MTM resonators were created by machining out two brass plates. Coupling to the negative index mode was non-trivial since the electric field topology of the negative index mode in the MTMBWO is unlike the fundamental waveguide mode. Several coupling designs were investigated to try and minimize reflection near the operation frequency. In this test design, coupling was accomplished through WR284 waveguide mated to the side of the MTMBWO test structure. This matched the polarization of electric field in the fundamental TE mode of the WR284 waveguide and the polarization of the axial field in the negative index TM-like mode of the MTMBWO test structure. This coupling scheme is different than that used in the PIC simulations of the MTMBWO, and was chosen for simplicity of design. Coax to WR284 waveguide adapters are used to excite the fundamental TE mode in the WR284 waveguide. The transmission measurement is shown in Fig. 13 in blue, along with a simulation of the same setup using CST Microwave Studio in red. The planned operation frequency band is highlighted in light blue.

Fig. 13. Transmission measurement of the test MTMBWO structure (blue) compared with CST Microwave Studio simulation (red). The planned operation frequency band is highlighted in light blue.

VI. CONCLUSION

The design of a microwave generator using MTMs in a negative index waveguide has been presented. HFSS eigenmode simulations have been presented to confirm the presence of the negative index mode and estimate coupling to the beam. The device operates as a backward wave oscillator with the negative index mode supported by the MTM plates. CST PIC simulations have been used to demonstrate the performance of the device in use with a 500 keV and 60-100 A electron beam. Efficiency of >14% is observed with saturated output at 5.75 MW. An effective medium model is presented which models the MTM as a dielectric with a Lorentzian response. The model structure is also simulated with the CST PIC code and the output power and frequency are in qualitative agreement with the full MTM PIC simulations. Finally, to verify the electromagnetic design of the MTM device a test structure was designed and tested using a VNA, and transmission measurements confirmed the presence and excitation of the negative index mode.

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