The effects of Finite electron temperature and Diffraction and on Lower Hybrid Waves

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The effects of Finite electron temperature and Diffraction and on Lower Hybrid Waves

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Abstract

In this paper we show that the commonly used cold plasma dispersion relation for plasma waves in the lower hybrid range of frequencies (LHRF) produces a wave trajectory that is notably different than when thermal corrections to the Hermitian part of the dielectric are retained. This is in contrast to the common implementation in which thermal effects are retained only for the anti-Hermitian part of the dielectric used.
for damping calculations. We show which term is the critical one to retain in the dielectric and discuss implications for modeling of LHRF waves in present day and future devices. We conclude with some observations on the effects of diffraction that may be isolated once thermal effects are retained in both ray tracing and full wave approaches.

1 Introduction

In theory and modeling of lower hybrid (LH) waves it is conventional to use the cold plasma model for propagation and only to invoke finite electron temperature effects in electron Landau damping \[3, 4\]. The origin of this convention is difficult to trace as it is not supported by the theory of LH current drive. The original proposed purpose of lower hybrid waves was as a method of heating ions through mode conversion \[12, 13\] of the electrostatic Bernstein wave to the fast electromagnetic wave at the lower hybrid resonance. This scenario permitted the neglection of the contribution of terms lower order in the perpendicular wave number. We shall show that the assumptions in this physical situation do not apply to the use of the cold plasma dispersion relation to describe lower hybrid ray trajectories in current drive scenarios and can miss important thermal corrections when the LH ray is launched directly.

Lower hybrid waves have been simulated using geometric optics (ray tracing), higher order WKB (beam tracing), and physical optics (full wave). Discrepancies \[1, 7, 10\] between the results of these approaches have been attributed to diffraction and focusing. We shall show that while these effects remain, primarily the differences observed were due to different dispersion relation models in those codes.

In the first part of this paper, we will show that full kinetic effects of finite temperature electrons must be retained in the parallel contributions to the dielectric to obtain accurate propagation as well as damping. Counter-intuitively, the thermal corrections are not important for the high temperature \(T_e > 10\) keV fusion reactor plasmas. The effect is observed if a given family of rays are sufficiently weakly damped that they encircle the magnetic axis and so are in the ‘multi-pass’ regime. We will quantify this effect for some simple cases and show that it is present for the parameters of present day tokamaks.

The use of ray tracing to determine trajectories relies on a small wavelength compared to system scale lengths and is accurate in determining the direction of energy propagation until focusing and thus diffraction begins to play a role. In the second part of this paper, we show that the flux averaged
power densities and current densities predicted will be higher than if these 
effects were accounted for.

2 Background

Lower hybrid (LH) waves were proposed [13] for ion heating through mode 
conversion to ion Bernstein waves and the lower hybrid resonance, \( \omega = \omega_{\text{lh}} \)
where

\[
\omega_{\text{lh}} \equiv \left[ \left( \Omega_{ce} \Omega_{ci} \right)^{-1} + \omega_{pi}^{-2} \right]^{-1/2} \approx \sqrt{\Omega_{ce} \Omega_{ci}}.
\]

Thermal effects were believed to play a role in propagation only through the finite value of \( k_\perp \rho_i \) near resonance and \( k_\parallel v_{te}/\omega \) in Landau damping and not in parallel dispersion. For the electrostatic limit that occurs near the LH resonance, a dispersion relation sixth order in the perpendicular wave number may be derived [2].

The cold plasma dispersion relation,

\[
P_4 n_\perp^4 + P_2 n_\perp^2 + P_0 = 0,
\]
describes a slow electrostatic mode and a fast electromagnetic mode. The parallel and perpendicular indices of refraction are given by \( n_\parallel, \perp = k_\parallel, \perp \omega/c \) where \( k \) is the wave number, \( c \) is the speed of light, and \( \omega \) is the wave frequency in radians/s. In magnetically confined fusion plasmas where magnetic fields are on the order of several Tesla, \( \omega_{\text{lh}} \) will be on the order of several gigaHertz. The coefficients are defined in terms of the Stix [12] cold plasma dielectric tensor:

\[
P_4 = S
\]
\[
P_2 = (S + P)(n_\parallel^2 - S) + D^2
\]
\[
P_0 = P \left[ (n_\parallel^2 - S)^2 - D^2 \right],
\]

as derived in Bonoli [2].

Bonoli [2] further derived the leading thermal corrections to Eq. (2) for the case where \( n_\perp \gg n_\parallel \) which is appropriate for waves near the hybrid resonance where \( S \to 0 \) and mode-conversion occurs. This correction resolves the resulting singularity by introducing a higher order term, \( n_\perp^6 \), with a coefficient of \( P_6 = (-3/2) \beta_i \Omega_{ci}^2/\omega^2 + (3/8) \beta_e \Omega_{ce}^2/\omega^2 \) that is proportional to the plasma beta, \( \beta \). Near the lower hybrid resonance, the \( n_\perp^6 \) term dominates the dispersion and thermal effects in the lower order terms may be neglected.
Modern usage of LH waves is for current drive and not ion heating and in this regime in which $\omega > 2\omega_{lh}$ to avoid edge parametric decay modes (reference?), the lower hybrid resonance is not in the plasma and so the thermal corrections just discussed are not included. Motivated by observed discrepancies in previous LHRF modeling benchmarks \[1, 7\], we have found significant differences in predicted trajectories of LH paths from raytracing between the cold plasma model and the full hot plasma dielectric.

3 Hot electron effects

When calculating the trajectories of low hybrid waves, it has been assumed that the paths are independent of electron temperature and the cold plasma dispersion relation may be used. In Fig. 1, we test this assumption by using the full hot dispersion relation including all cyclotron harmonics and thermal effects \[12\] in the GENRAY code (id=6 option labeled in the legend.) We compare rays for various electron temperatures to the cold plasma prediction (id=2.) The hot plasma rays converge to the cold plasma rays as expected but large differences in the poloidal location of the first reflection remain even at temperatures as low as 2 keV. The differences increase.

To further isolate the source of this additional dispersion, we consider what terms are important in the LHRF. Because of the high frequency relative to the ion gyrofrequency the ion harmonic terms should not be important. Similarly, $\omega_{lh}$ is far below the electron gyrofrequency and so the electron harmonics may be neglected. The largest Landau term is the Stix parallel dielectric, $P$, which is proportional to the electron and ion plasma frequencies. The electron term is dominant, so we consider the thermal corrections to that term. In Eqs. 1, the cold plasma expression (Eq. 1a), the first order in $k_{||}^2 v_{th}^2/\omega^2$ (Eq. 1b) and third order corrections (Eq. 1c) to parallel dielectric term, $P$, are given.

\[
P = 1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{pi}^2}{\omega^2} \quad (1a)
\]

\[
P = 1 - \frac{\omega_{pe}^2}{\omega^2} \left( 1 + \frac{3}{2} \frac{k_{||}^2 v_{th}^2}{\omega^2} \right) - \frac{\omega_{pi}^2}{\omega^2} \quad (1b)
\]

\[
P = 1 - \frac{\omega_{pe}^2}{\omega^2} \left( 1 + \frac{3}{2} \frac{k_{||}^2 v_{th}^2}{\omega^2} + \frac{15}{4} \frac{k_{||}^4 v_{th}^4}{\omega^4} + \frac{105}{8} \frac{k_{||}^6 v_{th}^6}{\omega^6} \right) - \frac{\omega_{pi}^2}{\omega^2} \quad (1c)
\]

which are just successive terms in an asymptotic series expansion of the full
Figure 1: Full hot plasma dispersion relation in GENRAY tends to the cold plasma prediction with decreasing electron temperature. Plasma parameters are $n_e = 5 \times 10^{19} \text{m}^{-3}$, $B_0 = 8$ Tesla, $n_\parallel = -2.5$, $a=22$cm, $R_0=75$cm.

kinetic expression involving the plasma dispersion function, $Z(\xi)$:

$$P(\xi) = 1 - \frac{\omega_{pe}^2}{\omega^2} \xi^2 Z''(\xi) - \frac{\omega_{pi}^2}{\omega^2},$$

where $\xi = \omega/(k_\parallel v_{th})$. The parameter in the thermal correction, $\xi$, is the wave phase velocity normalized to the electron thermal velocity and is typically a large number ($\sim 4-6$) such that perhaps Eq. 1b or Eq. 1c might be sufficient to capture the thermal effects. But numerical tests using ray tracing show that after the first reflection even the third order correction begins to noticeably depart from the full hot plasma dielectric. The same numerical tests show that using the plasma dispersion function correction to
the Landau term of $P(\xi)$ as in Eq. 2 is sufficient to capture all the dispersion of waves in the LHRF from the full hot plasma dispersion relation.

Additional poloidal effects introduced in $P(\xi)$ through the thermal correction lead to changes in $k_\parallel$ and $z$ evolution. Consider the cold plasma electrostatic slow wave dispersion relation, $D = S k_\perp^2 - P k_\parallel^2 = 0$. The evolution of the parallel position, $z$, of the ray versus time, $t$, is given by $\partial z/\partial t = -(\partial D/\partial k_\parallel)/(\partial D/\partial \omega)$. We find

$$\frac{\partial D}{\partial k_\parallel} = -2P(\xi)k_\parallel + \frac{\omega}{v_{te}} \frac{\partial P(\xi)}{\partial \xi},$$

with second term being new. This new term is primarily responsible for the change in the ray trajectory seen, although there are additional contributions to evolution in the flux, $\psi$, component of the wave number due to the temperature dependence, $T_e(\psi)$ of $\xi$ that have not been shown. Note that after substituting for $\xi$, the leading order behavior of the new term is proportional to $v_{te}$ and so vanishes as $T_e \rightarrow 0$.

### 3.1 Implementation in various codes

TORLH [16, 17] implements the kinetic form of $P(\xi)$ in the LHRF limit as given in Eq. 2. The LHEAF code [7, 10] uses the cold plasma approximation for $P(\xi)$ and iteratively corrects the imaginary part of $P(\xi)$ for damping but propagation is still determined by cold plasma dispersion. LHBEAM [1] also uses cold plasma dispersion for propagation and hot plasma dielectric for damping in results reported up to the publication of the paper but has been modified to include these new effects. The assumption of cold plasma dispersion for propagation is common enough that authors often do not describe the dielectric model in use in papers on LH waves. The cold plasma approximation is commonly used in ray tracing codes [5, 6]. Some codes, such as GENRAY [11] and C3PO [9] have options for cold and hot (kinetic) plasma dielectric models for LH but the cold dielectric is still the primary model used by users [4] and in published benchmarks [9].

### 3.2 Comparison of effects of rays for cold and hot plasma

In cases of strong absorption we have quantified the path differences. In Fig. 3 we compare the flux averaged power deposition from ray tracing to full wave. In this comparison for the ray tracing results we use multiple

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1 Personal communication with several users
launched rays covering the spatial extent of the launch height to capture the finite width of the LH beam. Using a single ray produces flux averaged power deposition profiles that are narrower that full-wave or beam tracing owing to the geometric projection of the lower hybrid beam onto the flux surface [8]. A single ray does not account for finite beam width when the ray path is not normal to the flux surface. Using multiple rays that cover the spatial and spectral width of the launched waves does capture the width of the beam the its power deposition. The ray bundle is shown in Fig. 5. The power depositions all agree well until the waves are with about normalize minor radial, $r/a$, of 0.1. Near the axis the full wave deposition has two well resolved peaks in the power, the hot dispersion ray tracing also has two peaks in the same locations but much more narrow and the cold dispersion ray tracing only has a single peak.

The difference in the power deposition seen in ray tracing with and without the finite electron temperature effect can be understood by referring back to Fig. 2. In that figure, the cold ray follows a trajectory that produces a higher upshift in $k_\parallel$ and results in damping of the ray before a second pass to the magnetic axis is complete. The hot ray reaches the axes and second time and produces the double peaked power deposition that is also seen in the full wave result.

In weak absorption the LH waves undergo many reflects and so one might expect the aggregate effect of finite electron temperature to be larger. The actual effect is that the rays become stochastic and space filling and trajectory differences have no effect on power deposition. In a multi-pass scenario with $n_\parallel \sqrt{T_e} < 5.7$ the LH waves are space filling. Thermal effects on on the power deposition calculated with ray tracing shows differences in peak locations but not in the width of the overall profile, this is shown if Fig. 4. Because of the multiple reflections of rays, it is not possible to attribute physical effects to specific features as it was in the single pass case. In comparison with full-wave, the power deposition from ray tracing is narrower even with both full-wave and ray tracing using the same hot plasma dispersion relation. We attribute this difference to two causes. In the ray tracing case, the rays have a maximum length over which they are traced. This results in incomplete power absorption and in this case only 65W out of 250W were absorbed. (It should be mentioned that normally such a weak absorption case would be iterated with Fokker-Planck evolution of the electron distribution until nearly complete absorption had been achieved.) In full-wave, all power is absorbed by construction. The second cause is diffraction and the resultant spectral broadening is included in the full-wave model. This is discussed further in Section 4.1 on diffraction effects.
Figure 2: Finite electron temperature affects the ray trajectory. A single ray is propagated with and without the inclusion of finite electron temperature effects. They are overlaid on contours of the magnitude of the parallel electric field from the full-wave solver TORLH. Parameters are $B_0 = 8$ Tesla, $n_e = 5 \times 10^{19} m^{-3}$, $n_\parallel = -2.5$, and $T_{e0} = 5$ kev.

In high density LH experiments in Alcator C-Mod [14], accessibility of LH power to the core plasma has been limited. One candidate for this observed effect is the trajectory of the rays into the scrape-off region loosing power to collisional damping. This process is dependent on the path of the rays and therefore the finite electron temperature effect under discussion in this paper may play a role.

4 Diffraction effects

4.1 Focussing

There are still differences between the power profiles from ray tracing and full-wave after finite width is taken into account. Investigating further, we see in Figure 5 that focusing in ray tracing is much tighter than in full-wave. Having accounted for the other differences in the models: finite electron temperature and finite waveguide height and spectrum, we can finally ascribe these observed differences to physical optics effects, namely diffractional broadening at beam foci.

In a previous paper [1] the authors compared ray tracing and the paraxial
beam approximation for the LH propagation in model equilibrium under discussion. We extend that comparison here with full-wave calculation that enables us to include reflections in the field calculations. We also employ a bundle of rays that have similar spectral and spatial widths to the antenna model in the full-wave code, TORLH. The poloidal width $k_\parallel$ spectrum from the full-wave code is used in the ray tracing code by having 8 rays launched with differing poloidal components covering the same width. Similarly, rays are launch along the physical height of the antenna. In this way, we have very similar boundary/initial conditions used in both full-wave and ray tracing.

In Figure 5, we see that the ray paths overlaid on the full-wave contours of electric field magnitude follow each other closely until the waves turn near the magnetic axis (the origin in this plot.) At that point, the rays continue to focus while the full-wave fields reach a minimum beam width and actually begin to broaden again. Following the rays and beam further as they near the wall and reflect the wave fields experience constructive and destructive interference then reflect and make a second pass to the center where they both damp further creating the second peak seen in Fig. 6 at a normalized radius of 0.06.
Figure 4: Finite electron temperature on power profile in multipass regime. Flux averaged power deposition from ray tracing and full wave simulations. In a multipass scenario with $n_{\parallel} \sqrt{T_e} < 5.7$ the LH waves are space filling. Vertical axis was truncated from 25 to 5.

4.2 Power deposition

After accounting for warm plasma effects we get much better overall agreement between the ray tracing a full-wave power deposition than in Fig. 8 of reference [1]. This permits this isolation of the diffraction effects. Referring to Fig. 6, we see that both models predict the double peaked aspect of the power resulting from the reflection. Accounting for the finite beam width results in a better match between power profiles outside of a normalized radius of 0.1 and also good agreement in the on axis power density. The notable difference is the width and height of those two peaks. Recalling that the one-dimensional power deposition results from an average over the the poloidal dimension, the consequence of focusing the power to a point in the poloidal plane as opposed to a finite spot size becomes evident. A finite width will spread the power across several flux surfaces whereas a point will always fall on just one. This ray tracing produces one dimensional power depositions that have narrower features and higher power densities the physical optics would predict. We note though, that this effect is minimized if the wave
Figure 5: A ray bundle propagated using a finite electron temperature. Same plasma parameters as in Fig. 3 but with 36 total rays spread spectrally equivalent to 1023 poloidal mode numbers and spatially across a 6.4 cm high waveguide. The full-wave simulation used 1000 radial cubic finite elements and 1023 poloidal modes.

fronts are mostly parallel to the flux surfaces in the poloidal plane, that is propagation oblique to the flux surfaces. In this case there is no difference between a finite spot size and a point as far as the flux surface is concerned. This case arises primarily under strong single pass absorption where the wave does not penetrate to the center of the device, for example, in LH scenarios in the ITER device.

5 Conclusions

In this paper we have shown that finite electron temperature effects have a significant effect on the propagation of lower hybrid (LH) waves and should be included in the real part of the dielectric as well and the imaginary part. We further identified the relevant term as the Landau component of the parallel dielectric, $P(\xi)$. The effect on flux averaged power is minor because of the exponential flux surface dependence of the electron Landau damping. However, the effect on propagation trajectory is significant and could be important when other damping sources play are role; for example: collisional damping in the cold edge at high densities [15].

Inclusion of this term in ray tracing, beam-tracing, and full-wave codes is recommended especially in modern tokamaks with modest or weak damping ($n_\| \sqrt{T_e} < 5.7$). The ITER tokamak will operator at high electron temper-
Figure 6: Flux averaged power deposition from ray tracing and full-wave simulations. The effects of diffraction near focal points near inside a normalized flux radius of 0.1 produce broader peaks in deposition in full-wave results. Simulation parameters are the same as in Figure x. GENRAY ray tracing results are for a finite spread of 36 rays.

atures \( (T_{e0} > 15 \text{ keV}) \) with a parallel wave number, \( n_{\|} = 2.5 \) and will be in a strong single pass regime in which the LH waves will damp fully at approximately normalized radius of 0.7. In this case, the short propagation distance mainly normal to the flux surfaces does not produce notable differences between hot and cold dispersion relations. Simulations not shown in this paper demonstrate that the power deposition location and shape is not changed to better than 1% and so for scenario development on ITER, the cold plasma model is sufficient. After accounting for electron thermal effects, a careful comparison between full-wave and ray tracing was done to isolate diffraction effects. It was shown that the main effect was to limit focussing and modestly broaden the power deposition profile and reduce the peak magnitude of the deposition relative to ray tracing.

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7 References

References


