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On the formation and stability of long-lived impurity-ion snakes in Alcator C-Mod


$^1$Princeton Plasma Physics Laboratory, Princeton, NJ, 08540, USA
$^2$MIT - Plasma Science and Fusion Center, Cambridge, MA, 02139, USA
$^3$MIT - Laboratory for Nuclear Science, Cambridge, MA, 02139 USA
$^4$University of California at Los Angeles, Los Angeles, CA 90024

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Plasma Science and Fusion Center
Massachusetts Institute of Technology
Cambridge MA 02139 USA

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On the formation and stability of long-lived impurity-ion snakes in Alcator C-Mod

L. Delgado-Aparicio$^{1,2}$, L. Sugiyama$^3$, R. Granetz$^2$, D. Gates$^1$, J. Rice$^2$, M. L. Reinke$^2$, W. Bergerson$^{4,2}$, M. Bitter$^1$, D. L. Brower$^4$, E. Fredrickson$^1$, C. Gao$^2$, M. Greenwald$^2$, K. Hill$^1$, A. Hubbard$^2$, J. Irby$^2$, J. W. Hughes$^2$, E. Marmar$^2$, N. Pablant$^1$, S. Scott$^1$, R. Wilson$^1$, S. Wolfe$^2$ and S. Wukitch$^2$

$^1$Princeton Plasma Physics Laboratory, Princeton, NJ, 08540, USA
$^2$MIT - Plasma Science and Fusion Center, Cambridge, MA, 02139, USA
$^3$MIT - Laboratory for Nuclear Science, Cambridge, MA, 02139 USA
$^4$University of California at Los Angeles, Los Angeles, CA 90024

E-mail: ldelgado@pppl.gov

Abstract. Long-lived (1,1) ‘snake’ modes were discovered nearly three decades ago, but basic questions regarding their formation, stability, and superb particle confinement - shown by surviving tens to hundreds of sawtooth cycles - have remained unanswered. High-resolution spectroscopic imaging diagnostics permit studies of heavy-impurity-ion snakes with unprecedented temporal and spatial resolution, making it possible to positively identify the SXR signals with specific ion charge states and to infer, for the first time, the perturbed impurity density, $Z_{\text{eff}}$, and resistivity at the center of these long-lived helical modes. The results show a new scenario for the formation of heavy-impurity-ion snakes, which can begin as a broad 1/1 kink asymmetry of the central impurity ion density, that grows and undergoes a seamless transition to a large crescent-shaped helical island-like structure inside $q < 1$, with a regularly sawtoothing core. This type of formation departs strongly from the nonlinear island model based on a modified Rutherford equation (MRE) proposed originally to describe the pellet-induced snakes and expanded further to account for the impurity effects (e.g. $\tilde{P}_{\text{rad}}$ and $\tilde{Z}_{\text{eff}}$). These new high-resolution observations show details of their evolution and the accompanying sawtooth oscillations that suggest important differences between the density and temperature dynamics, ruling out a purely pressure-driven process. Instead, many features arise naturally from nonlinear interactions in a 3D MHD model that separately evolves the plasma density and temperature.
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1. Introduction and motivation.

The presence of magnetohydrodynamic (MHD) perturbations from equilibrium often limit the performance of magnetically confined fusion (MCF) plasmas leading to a global confinement degradation which will deteriorate the conditions needed to achieve and sustain a burning plasma. Understanding the formation and stability of three dimensional (3D) helical modes in the core of an otherwise, axisymmetric toroidal configuration, is one of the challenges of current fusion research [1]. One of the most interesting and commonly observed examples of 3D magnetohydrodynamic (MHD) activity in the core of a magnetically confined fusion plasma is the long-lived helical mode found at the Joint European Torus [2]-[4] more than 25 years ago. The typical snake-like helical patterns observed are characterized by a small region of localized and enhanced plasma density that rotates within the field of view of various diagnostics and is radially concentrated on, or inside the $q = 1$ magnetic surface. These long-lived modes are closely associated with sawtooth oscillations of the $q \leq 1$ region and understanding them is potentially important for a burning plasma like ITER where the $q = 1$ radius can be as large as half the minor radius.

The standard model [3] to describe the snake formation is based on the physics of tearing modes and suggests that the localized cooling of the $q = 1$ surface by a high-speed frozen deuterium fueling pellet is responsible for an increase of the local plasma resistivity, which in turn causes a drop in the current density ($J_{\psi} \sim E_{\psi}/\eta$) that leads to the formation of a magnetic island with poloidal and toroidal harmonics $m = 1, n = 1$. This island traps the excess ions from the pellet and form the snake. The plasma is assumed to recover rapidly from the usual $\sim 5 - 10\%$ temperature perturbation that originated the mode formation, leaving behind a snake-like helical electron and impurity density perturbation which is easily observed with the soft x-ray (SXR) cameras as shown in Fig. 1. The long-lived snake perturbation is typically localized to within $\sim 10 - 50\%$ of the poloidal circumference, has a radial width somewhat smaller than its poloidal extent ($l_r < l_\theta$), and rotates with the plasma at speeds of few tens of km/resulting in typical frequencies of 1-20 kHz. It is also clear from Fig. 1 that these long-lived modes are surprisingly stable since they survive tens of sawtooth crashes with minimal impact on their amplitude or periodicity. The transient shrinkage of the location of maximum SXR brightness (which is commonly used as a proxy for the location of the $q = 1$ surface) as well as the slowdown of toroidal rotation due to the crash are also important attributes to be considered. The radial and poloidal structure of the snake as well as its rotation frequency is a common feature observed in multiple devices: JET [4], Alcator C [5, 6], JT-60 [7], ASDEX-Upgrade [8], Tore Supra [9], CDX-U [10] and HL-1M [11] for naming few examples.

Snakes have been a common feature in every major tokamak fusion experiment, as well as in spherical tori and reversed field pinches. A second type, produced by an accumulation of impurity ions rather than the deuterium ions from injected fueling pellets, is also observed. Impurity snakes probably appeared first during core impurity...
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Figure 1. (Color online) Time history of raw SXR brightness profiles of the impurity-ion-snake imaged with a vertical x-ray camera during ~60 toroidal transits around the torus. Three observed sawtooth crashes are indicated with white arrows.

accumulation in “type O” discharges in Doublet-III [12] and ohmic plasmas in PBX [13]; these modes were also observed in laser-blow-off experiments in PLT [14] and later examples include JET [4] and ASDEX [8]. Impurity snakes tend to have significantly lower ion densities than pellet snakes. A minimum critical excess charge density, or perhaps mass density, appears to be essential. Attempts to induce snakes with lighter impurities have been mostly unsuccessful [4] which will suggest that ion mass plays a fundamental role through $\rho d\vec{v}/dt + \nabla p \approx \vec{J} \times \vec{B}$, where $\rho \equiv \sum n_i m_i$ is the mass density and $p \approx (n_e + n_D + n_Z) k_B T$ is the plasma pressure. This condition will suggest that impurity snakes might be possible in ITER as in the case of ADEX [8]. Both types of snakes possess surprisingly good MHD stability and particle confinement, since they can survive tens to hundreds of sawtooth cycles. As such, they provide important clues for understanding and controlling sawteeth in fusion burning plasmas.

Nearly three decades have passed since their discovery and observation in every major fusion facility; however, the details of its formation and stability have continued to puzzle plasma physicists worldwide. Undeniably, some of the most common questions regarding snakes involve: a) the details of the mode formation; it is still not clear if it follows an internal kink or a tearing mode description, b) the reasons behind its improved stability at a nearly constant saturated island width, c) whether the snake can be considered as 3D equilibria, d) the observed reduced particle transport in the center of the island in comparison to that of the anomalous transport dominating the background plasma, e) the role of the enhanced impurity density, plasma pressure and resistivity in the center of the island with respect to that of the background plasma, f) its resilience to tens or even hundreds of sawtooth cycles and crashes, and g) its impact or relationship with the background plasma [e.g. interaction of the (1/1) modes with
fast ions]. The difficulties on addressing such topics lie on the theoretical challenges of modeling core plasma perturbations as well as the experimental complications involved in diagnosing the center of the plasma with adequate spatial and time resolution, as well as acceptable signal-to-noise ratios. This paper reports new experimental data on the heavy impurity snakes in the Alcator C-Mod experiment [15], which leads to a new picture of its formation and sustainment, as well as its complex relationship with the sawtooth instability. A suite of novel spectroscopic imaging diagnostics has facilitated the identification of the major impurity ion species and the determination of the perturbed radiated power density inside the $q \leq 1$ region with unprecedented temporal and spatial resolution. For the first time, it is possible to infer the impurity ion density, electron temperature, $Z_{\text{eff}}$, and resistivity of the $n = 1$ helical structure [16]-[22]. These results can also shed light on the formation and stability of pellet-induced snakes.

This paper is arranged as follows: In Sections 2 and 3 we discuss the diagnostic tools used in Alcator C-Mod as well as describe the details of the dynamic evolution of impurity snakes and their complex relationship with the sawtooth instability. The theoretical implications of the current observations and the description of a novel 3D MHD model that separately evolves the density from the temperature dynamics is discussed in Sections 4 and 5, respectively; the summary and future work can be found in Section 6.

2. Diagnostics used for the characterization of the long-lived snakes.

A top-down view of the diagnostic set installed in Alcator C-Mod and used in this research is depicted in Fig. 2-a). Also indicated in the latter are the inner and outer mid-plane limiters at $R = 0.44$ m (by the shaded area) and $R = 0.905$ m (by the outer dotted line), respectively, along with the vacuum chamber wall at $R = 1.05$ m; the typical directions for plasma current, toroidal velocity and ambient field are also indicated. The x-ray tomographic (XTOMO) system [23, 24] used extensively in this paper is located in between ports C and D and the distribution of its integrating sightlines lie within a single poloidal plane as depicted in Fig. 2-b); the chords from the XTOMO vertical and horizontal systems used to identify the snake location appear highlighted and numbered. The SXR diodes are filtered using 50 $\mu$m Be foils, with a characteristic 50% transmission at $\sim$2 keV, allowing them to observe the emission from both intrinsic and extrinsic impurities. The SXR preamp voltages are digitized at $\sim$250 kHz which is enough to resolve the snake since its toroidal frequency is of the order of few kHz. The x-ray tomographic reconstructions are obtained at a minimum time step of 4 $\mu$s, using a basis of 15 radial Bessel harmonics [25], a singular value decomposition (SVD) tolerance of 0.1, and a 48 by 84 cm horizontal and vertical emissivity grid with a typical spacing of 1 cm [18, 19]; the spatial and poloidal resolution are of the order of 1-2 cm and 10-20$\circ$.

Unfiltered silicon diodes are also being used in Alcator C-Mod to measure two
dimensional structure of the radiated power density profiles in order to investigate poloidal asymmetries in the core radiation [26, 27]. Two 22-channel Absolute eXtreme UltraViolet (AXUV- 22EL, see ref. [28]) diode arrays view the plasma tangentially 0.5 cm above the equatorial midplane in two different toroidal directions as shown in Fig. 2-a). One array, AXA, observes the outboard half of the plasma with a spatial resolution of $\sim$ 1 cm. The other array, AXJ, views the full midplane profile but looks in the opposite toroidal direction and has a spatial resolution of $\sim$ 2 cm. Photocurrents are fed into standard transimpedance amplifiers with gains on the order of $10^4$ V/A and a bandwidth just over 100 kHz. The local emissivities reported in this paper have been computed using a common discretized Abel-inversion method that is complemented with a linear regularization technique [26, 27]; from the time and space variations of the local emissivities it is possible to infer a measurement of the snake electron and impurity density from $\delta n_Z \simeq \delta P_{rad}/n_{e,0}L_Z$ and $\delta n_e \simeq Z\delta n_Z$, where $L_Z$ is the cooling rate for the impurity of choice and $\delta P_{rad}$ is the net radiated power emitted by the snake. The $n = 1$ nature of the density perturbation has been measured for the first time using these two toroidally displaced AXUV arrays. Unlike most previous observations, the C-Mod SXR and $P_{rad}$ signals separate the ion density effects from those of electron temperature. Fluctuations in the electron temperature are measured separately using the high resolution electron cyclotron emission (ECE) radiometer [29, 30] installed 90°
apart in toroidal angle from the X TOMO system and nearly 180° apart from the AXA array [see Fig. 2-a]. This broadband heterodyne radiometer measures second harmonic ECE at 234-306 GHz and its functional for typical machine operation at $B_T = 5.4$ T. This diagnostic measures $T_e$ on the low field side with 32 channels separated by <1 cm and a frequency response of 1 MHz.

The identification and localization of the main impurity species was possible using vacuum ultraviolet (VUV) and SXR spectroscopy. VUV spectroscopy is performed using a 2.2 m, grazing-incidence Rowland circle McPherson spectrometer using a 600 l/mm grating [31]. Although a wide wavelength region of 90-1030 Å can be scanned, only several tens of Å can be observed at a time with resolving power $\sim 100$. A reduced spectrum coverage can be obtained at a rate of up to 2 kHz [31]. The spectrometer views the plasma on a poloidal plane located in H-port [see Fig. 2-a)] with a single integrating line-of-sight. Space and time resolved SXR spectroscopy on intrinsic (e.g. Mo) and extrinsic (e.g. Ar) impurity ions is possible thanks to the operation of the High Resolution X-ray crystal imaging spectrometer with Spatial resolution (HiReX-Sr) installed at Alcator C-Mod as a collaborative effort between the MIT-PSFC and PPPL (see [32]-[35] and references therein). Although this diagnostic was designed primarily for extracting temporally and spatially-resolved spectra from argon, from which local ion and electron temperatures ($T_{i,e}$) and plasma flow velocity components ($v_{\phi,\theta}$) can be inferred, it is also possible to monitor intrinsic impurities such as molybdenum. Its diagnostic arrangement is depicted in Figure 3-a) and consists mainly of a set of two spherically bent crystals that serve as diffraction elements, together with two-dimensional position-sensitive x-ray detectors which record high resolution spectra from hundreds of sight lines that intersect the C-Mod poloidal cross section [installed in B-port - see Fig. 2-a)]. The spectrometer has two well-defined branches in order to record helium- and hydrogen-like argon spectra. Within the latter, we also identify a

![Figure 3.](image)

**Figure 3.** (Color online) a) Schematic view of the HiReX-Sr spectrometer installed in Alcator C-Mod. b) The Ne-like molybdenum (Mo$^{32+}$) emission line is next to the Ar Lyman-α doublet. This snapshot was recorded (at $t=0.9135$ s) with a 10 ms integration time, 400 ms after the Ar puff, while the snake was observed at $t \in [0.334, 0.420]$ s.
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strong neon-like molybdenum emission line (Mo$^{32+}$, see reference [36]) that falls right next to the typical Ar Lyman-α doublet as shown in Figure 3-b). The molybdenum and argon brightnesses show peak profiles with a characteristic argon Lyman-α ratio of approximately $\frac{L_y\alpha_2}{L_y\alpha_1} \sim 0.5 - 0.6$ [37].

Multi-chord Faraday-Effect measurements in C-Mod [installed in K-port - see Fig. 2-a)] provide internal measurements of the equilibrium poloidal magnetic field and fluctuations on the Faraday signal associated with snakes have been recently reported [38]. Preliminary results suggest the presence of magnetic perturbations away from the magnetic axis (measured also with flux loops at the wall), which may indicate that toroidal effects of the snake introduce also an $m = 2$ component extending from the $m = 1$ core region. Efforts are underway to differentiate between density and magnetic fluctuations in the polarimetry measurement via cross correlation techniques and combined density measurements [e.g. using the signals from a ten vertical-viewing CO$_2$ laser chords of a fast two-color interferometer (TCI, see [39]) installed in H-port - see Fig. 2-a)]. Thanks to the implementation of all these diagnostics with adequate spatial and time resolution, which supplement that of the the conventional broadband soft x-ray (SXR) tomography, it is now possible to study the role of the perturbed radiated power ($P_{rad}$), impurity density ($n_z$), $Z_{eff}$, resistivity ($\eta$), collisionality ($\nu$) and toroidal plasma flow velocity ($v_\phi$) in the formation and stability of snakes. Moreover, the enhanced plasma pressure and resistivity at the center of the impurity-induced snakes have now been measured for the first time, which in turn sheds light onto the stability mechanism needed to sustain such modes.

3. Dynamic evolution of the long lived snake.

The C-Mod snake formation can be described as a three step process corresponding to the numbers in Fig. 4. These stages describe the time history before the snake formation - during the core impurity accumulation, the kink-like circular distortion prior to the onset of sawtooth crashes - which can last from few to hundreds of milliseconds, and the helical crescent like perturbation which resembles that of a magnetic island, and which coexist with the sawtooth instability.

3.1. Before the snake formation

Helical snake modes like the ones shown in Fig. 4 have been observed in C-Mod ohmic discharges during the plasma current ramp-up phase or early in the plasma current flattop, where the high edge temperature increases the high-Z impurity erosion from the inner wall (e.g. molybdenum limiter), and the absence of sawtooth crashes allows on-axis impurity peaking. An example of their apparently spontaneous formation is depicted in Fig. 4 using frequency spectrograms and the time-history of the soft x-ray (SXR) brightness profiles. As mentioned above, the snake frequency during its formation and lifetime is $< 10$ kHz and the (1, 1) mode appears concentrated in the plasma core [see
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Figure 4. (Color online) Spectrograms and raw SXR brightness profiles of two impurity snakes a) and b); horizontal axes are time and the major radius labelled by vertical viewing chords. The dotted lines indicate the time of the snake formation while the white arrows in b) indicate the first two sawtooth crashes.

Figure 5. (Color online) Radiated power density profiles - during the core impurity accumulation of stage 1 - before the snake formation of the two snakes a) and b) shown in Fig. 4.

SXR coverage in Fig. 2-b)]. The second snake shown in Fig 4-b) had nearly identical radiation profiles until an inadvertent high-Z impurity injection at 0.324 s - 10 ms before
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Figure 6. (Color online) Spectra obtained with the Roland circle McPherson spectrometer - during stage 1 of core impurity accumulation - before the formation of the two snakes a) and b) shown in Fig. 4.

Figure 7. (Color online) a) Radial profiles for the molybdenum brightness measured with the HiReX-Sr spectrometer before the snake formation. The time history profile of a central-channel is depicted in b), and shows a remarkable agreement with the central brightness signatures measured using the vertical and horizontal XTOMO cameras.

the snake formation - nearly tripled its radiated power density \(P_{rad}\) at mid-radius while increasing the core emissivity by just 10% [compare Figs. 5-a) and -b)].

The VUV spectra obtained from the McPherson spectrometer during stage 1 of core impurity accumulation of the two snakes a) and b) shown in Fig. 4, is shown in Figs. 6-a) and -b), respectively. This data indicate that the brightness associated to the 3s-3p Na-like and 3s2-3s3p Mg-like molybdenum lines at 127.8 and 116.0 Å increase nearly by a factor of \(\times 4\) and \(\times 10\) during the impurity accumulation before the snake formation. The spectral coverage of this spectrometer ranges from 100-140 Å and includes the 2s\(^2\)-2s2p Be-like lines from Fe, Ni, and Cu; the later impurities are components of the stainless steel INCONEL present in the vacuum vessel, as well as parts of the ICRF antennae straps. These emission lines have been detected in various instances but were absent during the times of the snake and thus do not contribute to the increase in radiated power. Molybdenum is the main high-Z intrinsic impurity at C-Mod since more than
95% of the plasma facing components are made out of pure molybdenum and/or TZM, an alloy that contains 99% Mo, 0.5% Ti and 0.08% Zr. The average molybdenum ion charge for a core electron temperature of interest ($T_{e,0} \sim 1 - 3$ keV) is $\langle Z_{Mo} \rangle = 32$, with a broad ion fraction of $\sim 50\%$ for $Z = 32$, and $20\%$ and $30\%$ for $\text{Mo}^{31+}$ and $\text{Mo}^{33+}$, respectively (see references [36, 40]). The intensity of the SXR $\text{Mo}^{32+}$ brightness profiles - obtained with a reduced integration time of 10 ms - is shown in Figure 7-a), and indicate a slow peaking of molybdenum density before the snake formation. The normalized time history of a core-viewing sightline is depicted in Figure 7-b), and shows a remarkable agreement with the central normalized brightness measured with the XTOMO cameras [18]. Consequently, it is safe to assume that the net enhancement in the core emission before the formation of the snakes shown in Fig. 4 is strictly due to the presence of molybdenum charge states and that the snake-like pattern in the SXR data is formed by a small region of localized and enhanced molybdenum density on or inside the $q = 1$ surface.

Stage one, before the snake formation, is characterized therefore by an axisymmetric central peaking of the SXR and $P_{rad}$ profiles due to intrinsic molybdenum impurity accumulation [see red profiles for $t \approx 0.285$ s and $t \approx 0.334$ s in Figures 5-a) and -b), respectively]. For snake (a) shown in Figs. 4-5-a), the core electron density before the impurity accumulation ($t_0 \approx 0.140$ s, at the end of the current ramp-up phase) is flat.

![Figure 8.](image-url)
and \( n_{e0} \simeq 1.2 \times 10^{20} \text{ m}^{-3} \), while just before the snake formation \((t_1 \simeq 0.285 \text{ s})\) it peaks to \( n_{e0} \simeq 2.2 \times 10^{20} \text{ m}^{-3} \) [see also black and red profiles of electron density and temperature depicted in Fig. 8-a)]. The net core \( P_{\text{rad}} \simeq 2.1 \text{ MW/m}^3 \) then corresponds to a core Mo density \( n_{Mo} \simeq 1.4 \times 10^{17} \text{ m}^{-3} \) and a peaked impurity concentration \( n_{Mo}/n_e \simeq 6.3 \times 10^{-4}. \) The latter calculation uses a Mo cooling factor \( L_{Mo} \simeq 7 \times 10^{-32} \text{ W·m}^3 \) which appears to be nearly independent of the electron temperature in the range of 1-3 keV [40]; the core electron temperature is of the order of 1.5 keV. Such an impurity density can increase the core \( Z_{\text{eff}} \), as well as the collision frequency \( \nu_{ei} \) and resistivity \( \eta \), by more than 50% over the molybdenum-free state. Central values of radiated power density up to \( \simeq 3.6 \text{ MW/m}^3 \) have been observed in these discharges [see Fig. 5-b)] and thus even larger increments of \( Z_{\text{eff}}, \nu_{ei} \) and \( \eta \) could be expected when compared to molybdenum-free plasmas.

3.2. During the sawtooth-free kink-like snake formation

In the second stage, the snake forms as a growing and rotating kink-like \((m, n) = (1, 1)\) helical impurity density concentration with a nearly circular cross section, as shown in the \( P_{\text{rad}} \) profiles in Fig. 9 and the 2D SXR reconstructions in Fig. 10 [18, 19]. This broadly kinked snake precedes any sawtooth onset. The peak central radiated power density profiles increase by an additional \( \simeq 5-10\% \) (see Fig. 9), since the impurity ions are now confined to a smaller volume; this phenomenon was first identified by Stutman et al. as a vacuum cleaner effect in CDX-U [10]. The reconstructed midplane emissivity 500 \( \mu \text{s} \) before and after the toroidal and poloidal excursion are shown in Figs. 10-a) and -b), respectively; the 2D tomographic reconstructions obtained at the times indicated in Fig. 10-b) are depicted for completion in Fig. 10-c). The displaced circular core resembles now that of a low amplitude internal and ideal \((m, n) = (1, 1)\) kink rotating with the plasma toroidal flow velocity. From the net radiated power density and assumed

**Figure 9.** (Color online) Radiated power density profiles before and during the snake formation (stages 1 and 2) of the two modes a) and b) shown in Fig. 4.
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Figure 10. (Color online) Time history of the equatorial midplane SXR emissivity a) before and b) during the formation of the snake shown in Fig. 4-b). Each successive frame depicted in the 2D tomographic reconstruction shown in c) is 100 µs apart, as indicated also in b).

quasi-neutrality it is possible to infer the impurity density at the center of the snake: $\delta n_{Mo} \simeq \delta P_{rad}/n_{e,0}L_{Mo}$ and its associated electron density perturbation $\delta n_e \simeq Z\delta n_{Mo}$. The Thomson Scattering electron density and temperature profile measurements are unable to detect the snake since these fluctuations are of just few percent, while the typical error bars on the measurement are of the order of $\pm 10\%$. A net 2.7 MW/m$^3$ with a flat core $n_{e,0} \simeq 1.7 \times 10^{20}$ m$^{-3}$ will correspond to an increased snake density of $\delta n_{Mo} \simeq 2.2 \times 10^{17}$ m$^{-3}$ with $\delta n_e \simeq 7 \times 10^{18}$ m$^{-3}$. The perturbations in density and charge can be as high as $\delta n_e/n_{e,0} \simeq 4\%$ and $\delta Z_{eff}/Z_{eff,0} \simeq 60-70\%$ [18, 19].

An example of frequency spectrograms of SXR brightness, central electron temperature and $P_{rad}$ ($\propto n_{Mo}$) during the snake formation are depicted in Figs. 11-a), -b) and -c). The $m = 1$ component of the density perturbation during the formation of the kink-like distortion ($t \in [0.289, 0.291]$ s) is measured using two channels of the SXR vertical array labeled Ch. 10 (in black) and Ch. 16 (in red) shown in Fig. 11-d); these signals appear displaced $\Delta \theta \sim 180^\circ$ since they correspond to emission from the low-field side (LFS, $\theta \sim 0$ in black) and high-field side (HFS, $\theta \sim \pi$ in red), respectively. Typical $T_{e,0}$ oscillations measured with an electron cyclotron emission (ECE) radiometer at three midplane positions are shown in Fig. 11-e). The $n = 1$ component of the density perturbation has also been measured for the first time using two toroidally displaced AXUV arrays as shown in Fig. 11-f). The phase between the two signals is
nearly the same as the geometric angle between the arrays [see Fig. 2-a)]. These density and temperature fluctuations associated to the sawtooth-free part of the kinked-snake can last from few milliseconds to nearly half a second, depending on the conditions to destabilize the sawteeth.

3.3. Coexistence with sawtooth instability

The broad, almost circular kink then makes a seamless transition to the third stage, a 1/1 helical structure which resembles the crescent-shaped magnetic island produced by a resistive 1/1 internal kink; this form endures for the life of the snake. A sample of the 2D SXR tomographic reconstructions of the molybdenum island in between sawtooth crashes is shown in Fig. 12. The solid contours represent 10 to 90% of the maximum SXR emissivity; the position of the $q = 1$ rational surface inferred using the EFIT reconstruction code [41] is also indicated. For this third stage it is common to find approximately 20 snake periods between sawtooth crashes since their typical toroidal transit time is of the order of $t_{\text{snake}} \sim 185 - 200 \mu s$ while the sawtooth cycle lasts approximately $\tau_{\text{sawtooth}} \sim 3.6$ ms. The island structure in between sawtooth cycles remains roughly constant throughout the lifetime of the snake. These emissivity contours show a well localized region of stronger emissivity inside the location of the $q = 1$ surface, while the background emissivity is broad, suggesting also a flat core electron and impurity density profile. The temporally fluctuating SXR emissivity can be assumed to arise from a poloidally localized perturbation of the form
\[ \mathcal{E}(R, \theta, t) = \mathcal{E}_1 \exp\{-\left(\frac{r - r_s}{\omega_{sat}}\right)^2\} \exp\{-\left(\frac{\theta - \theta_s}{\sigma_{\theta}}\right)^2\} \] in which the intensity of the perturbation is dominated by the impurity density fluctuation (\(\mathcal{E}_1 \propto n_{e,0} \tilde{n}_{Mo}\)). The impurity accumulation at the center of the snake is such that its SXR emissivity is generally between three to four times bigger than that of the unperturbed core plasma, suggesting that the impurity density at the center of the snake is larger than that of the background core. The location \(r_s\) of the island is approximately 5 cm from the original core (\(r_s/a \approx 1/5\)), while its full-width at the base of the gaussian-like profile, as well as its poloidal coverage, are of the order of \(2\omega_{sat} \approx 6\) cm and \(\sigma_{\theta} > \pi\) radians, respectively; the relationship between the radial and poloidal extents (\(l_r \sim 6\) cm < \(l_{\theta} \sim 23\) cm) is also similar to various density-related snakes observed in various tokamaks [5]-[11].

The gradual reduction of impurity density can be inferred first, from the decrease of the intensity of the SXR brightness signals shown in Figure 13-a), -b) and -c). Both the SXR brightness of the background and snake decrease in time due to the combined effects of the sawtooth crashes and steady state transport. Such losses can also be inferred in detail from the 2D reconstructions shown in Figs. 13-d)-g). The peak emissivity decreased a factor two after several sawtooth cycles while the poloidal area covered by the crescent snake tends to get enlarged in time since \(q = 1\) surface is expanding radially outwards due to the radial inward diffusion of the plasma current density. An assessment of the changes in impurity density, resistivity and \(Z_{eff}\) can be done using the radiated power density profiles shown in Fig. 14-a) and -b), which were obtained when the snake crossed the equatorial midplane at the low-field-side (LFS, \(\theta \sim 0\)) and the high-field-side (HFS, \(\theta \sim \pi\)), respectively. Over 50 ms, the typical net radiated power density carried by the snake decreases by \(\simeq 1.0\) MW/m\(^3\) due to the combined effect of the background transport and sawtooth crashes [18, 19]. The inferred peak snake density decreases by
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Figure 13. (Color online) a)-c) Time history of the brightness profiles suggest a decrease in the impurity density of the mode and the typical gaussian-like background. The region of maximum emissivity also appears to be displaced in time to a different location radially outwards. d)-g) Time history of the 2D SXR emissivity profiles of the $m = 1$ molybdenum snake b) of Fig. 4. The solid contours in d) represent 10 to 90% of the maximum SXR emissivity of $\sim 1.6 \text{ MW/m}^3$.

$\simeq 50\%$, while the core electron temperature remains nearly constant at 1.65-1.7 keV [see inset in Fig. 14-c)]. The density perturbation narrows steadily and the location of the peak emission moves outwards in minor radius by $\delta r \simeq 2$ cm. The resultant enhanced resistivity at the center of the snake, in comparison to that of the background core, decreases from $\delta \eta/\eta_0 \sim \delta Z_{\text{eff}}/Z_{\text{eff},0} = 45\%$ to 20% [18, 19].

This last phase of the snake formation is characterize by a high impurity density crescent snake surrounding a smaller circular core of lower impurity density, but higher temperature; this helical crescent density is strongly resilient to sawtooth collapses as is shown also in Figs. 15-a), -b) and -c). During a typical sawtooth crash, the hotter circular core moves rapidly outwards to the edge of the crescent radius [see white arrow in Fig. 15-b)], where it coincides with an outgoing electron heat pulse. Meanwhile, the peak impurity density of the crescent snake remains nearly unperturbed showing
a decoupling between the dynamics of the temperature (pressure)-driven instability of the core and the snake density. The effect of the sawtooth crashes on the snake is to reduce, only transiently and by a small amount ($\sim 1 - 2$ cm), the location of the center of the (1,1) mode and possibly also, that of the $q = 1$ rational surface as suggested from recent experiment in ASDEX [42]. The snake toroidal transit frequency slows by $\simeq 25\%$ at the crash [periodicity changes from $\Delta t \sim 160\mu s$ to $\Delta t \sim 200\mu s$, see top of Fig. 15-a]), but later recovers, suggesting a transient change of the toroidal or poloidal flow velocities. The changes of radiated power density profiles associated with a sawtooth cycle are shown in Figure 15-d) and -e). A comparison of such profiles for times in-between two consecutive sawtooth crashes, and after approximately twenty toroidal transits is depicted first in 15-d); a similar observation right before ($\Delta t \sim -500\mu s$) and after ($\Delta t \sim +500\mu s$) a sawtooth crash is depicted in Figures 15-e). Within a sawtooth cycle, the center of the snake is shifted radially outwards by 1-2 cm, as was also noted

**Figure 14.** (Color online) Time evolution of the radiated power density during the last phase (stage 3) of snake (a) from Fig. 4-a). Figs. a) and b) were obtain at times were the (1,1) snake crossed the midplane at the LFS ($\theta \sim 0$) and HFS ($\theta \sim \pi$), respectively. The inferred impurity density perturbation is shown in c).
Figure 15. (Color online) a) Time plot of SXR brightness before and after a sawtooth crash in the stage 3 snake (viewing sightline intersects the outer edge of the snake). The SXR tomographic reconstructions b) and c) were done at the times indicated in a). Arrows show the direction of motion of the dark lower density (higher temperature) circular core. Changes of the radiated power density profiles during the sawtooth cycle and before and after the sawtooth crash, are shown in d) and e), respectively. The times chosen for the AXUV reconstructions are also indicated by the dotted-lines in a).

above in the SXR brightness contour plot shown in Fig. 1; the radiated power density at mid radius decreases \( \sim 100 \text{ kW/m}^3 \) while keeping the same radiated power at the center of the mode, which suggests that the underlying diffusion and convective transport is responsible for reducing the background impurity density. However, during a sawtooth crash, the center of the snake shrinks back (e.g. radially inwards) by the same amount of 1-2 cm. The radiated power density at the center of the snake decrease this time by 200 kW/m\(^3\), while the background profile remains invariant suggesting that each sawtooth crash contributes to flushing impurities from the core. The changes in ion-impurity and electron density due to a single sawtooth crash, and its cumulative effect during multiple sawtooth cycles will be addressed in a future contribution.
4. Theoretical implications

4.1. Limitations of the MRE formalism.

The impurity snake formation departs strongly from the nonlinear island model based on a modified Rutherford equation (MRE), proposed by Wesson [3] and subsequently applied to impurity snakes [9, 11]. A discussion of the MRE formalism applied to the impurity-ion-snakes has been included as an Appendix in Section 8. The MRE model postulates that a sudden excess of ions at $q = 1$ will induce a “nearly immediate” cooling of the rational surface, which will increase the local plasma resistivity, leading to the formation of a magnetic island. Instead, no evidence for an island is observed during the early C-Mod snake. Moreover, these snakes occur in plasmas with a slow impurity accumulation in the core, with no “instantaneous” cooling. Islands with $m \geq 2$, for which the MRE was derived, create only small displacements beyond their rational surface and can be described by local conditions. Such formalism assumes that $\Delta'$ [“jump” in derivative of the continuous magnetic flux function ($\psi$)], can be equated from a matching condition which arise from both sides of the rational surface (e.g. $q < m/n$ and $q > m/n$). The MRE model assumes therefore that only conditions at the $q = m/n$ rational surface affect island formation, but the only $m = 1$ instabilities known to produce a 1/1 magnetic island couple to the entire volume inside $q < 1$. Bussac et al. [43] showed for instance, that for the 1/1 internal kink in a torus, one requires consideration of the entire region $q < 1$, even for infinitesimal displacements. For impurity snakes, the local model also greatly over-emphasizes the destabilizing effects of the cooling of the $q = 1$ layer by line and continuum radiation ($P_{rad}$) as well as the change in ion charge ($Z_{eff}$), unlike the case of $m = 2$ magnetic islands [44]. This is shown by the large, destabilizing numbers obtained above. The saturation of the island, needed to create a sustained snake, then requires that the contribution from the surrounding equilibrium, measured by $\Delta'$, be large and stabilizing and remain so during the snake lifetime, a condition that appears to rule out the possibility of sawtooth oscillations.

4.2. Limitations of models based on ideal MHD equilibria

The conditions of reduced transport and enhanced plasma density over the background shown by the impurity-induced snake resembles also that of the geometry and stability of the 3D $m = 1$ quasi single-helicity (QSH) state observed in reversed field pinches (RFPs, see [48]-[51] and references therein). The formation of a stationary helical symmetric configuration in the RFP is due to a global single helical deformation of the current channel; similar changes have also been theorized to occur for the $m = 1$ snake in tokamaks as described analytically by Annibaldi [52] and Silveira [53]. New ideal MHD simulations have suggested also that the fully developed snakes can be due to

‡ Similar ideas of accumulation of impurities in $(2/1)$ tearing modes have been developed recently to explain the origin of the tokamak density limit scaling [45, 47].
a spontaneous equilibrium bifurcation that produces 3D helical structures in the plasma core while keeping axisymmetric boundary conditions. Using computationally intensive numerical simulations that describe tokamak equilibria, Garabedian [54] and Cooper [55] have found that, despite imposing axisymmetric boundary conditions at the edge of the plasma enforced by the geometry of the coil system, the preferred lowest energy state of MHD equilibrium can be non-axisymmetric in the plasma center. Numerical simulations of ideal magnetohydrodynamic equilibrium states with a 3D helical core have also been computed to reproduce some features of the original snakes observed at JET, as well as predict helical ITER equilibria possibly to be found during its future hybrid scenario (see [55] and references therein). A bifurcated set of equilibrium solutions is obtained using identical initial conditions except for the guess of the position of the magnetic axis. The energy difference between the axisymmetric and helical snake equilibrium is reported to be minimal.

Explanations of the snake based on static ideal MHD require, nonetheless, strong constraints on the safety-factor and pressure profiles. These 3D helical equilibrium states resemble qualitatively that of a saturated internal ideal kink mode and may be of interest only when studying the early sawtooth-free stages of pellet and/or impurity snake formation. However, observed gradual formation from zero amplitude, smooth transition from a broad kink to a crescent shape (very different from the helical core equilibrium), and the details of the periodic sawtooth crashes appear to rule out this type of explanation. The likely explanation for the persistence of the snake is that it embodies a quasi-steady-state magnetic structure. The snake’s 1/1 helical configuration and close association with the $q = 1$ surface and ongoing 1/1 sawtooth crashes suggest that it is related to the 1/1 internal kink mode [43] with magnetic reconnection.

Figure 16. (Color online) Expanded view of the time history of the SXR brightness of a helical molybdenum snake surviving a sawtooth crash without poloidal shifts or excursions [64]. The horizontal and vertical axes correspond to time and tangency radii; a 2D tomography reconstruction before the crash is shown next.
4.3. Limitations of MHD models based solely on pressure

The snake formation and its stability through sawtooth crashes are not easily explained by simple ideal MHD models. The long lifetime over energy and particle confinement times suggests the existence of a rather dynamic helical magnetic structure. As explained in this paper, both the radiated power density and SXR emissivity of the snake is enhanced over that of their background so it is common practice to associate the center of the snake with a region of enhanced density. The latter is the main reason why various MHD models, with 1/1 magnetic islands [56, 57] or without [55], generally equate the higher snake density also with higher pressure. Dynamically, a 1/1 island structure has been assumed to arise through one of the two known 1/1 instabilities, the ideal MHD internal kink [43, 58, 59] and its resistive variants or the quasi-interchange mode for \( q \approx 1 \) [60, 61]; saturated nonlinear 1/1 states [56, 61, 62] have also been proposed. These models have a fundamental difficulty in explaining the snake’s stable co-existence with periodic sawtooth crashes [63].

The internal kink-type instability that causes the crash is driven by the pressure gradient over the \( q < 1 \) region [43, 58]; nonlinearly, the crash displaces and destroys, or mostly destroys, the high-pressure circular core, thus reducing the drive for the instability. Additional heating of the core (e.g., during the early phase of the ohmic discharge or the rise phase of the sawtooth) would raise the pressure and destabilize the kink instability. The resulting crash of the circular core would destroy the snake density concentration along with the core pressure. As the core disappears, the snake density would have to rapidly flow around the \( q = 1 \) surface to the poloidally opposite side. In few words, these MHD models suggest that the snake, instead of remaining intact, breaks apart and then reforms, surviving the collapse; in such case, the position of the snake would have to experience a sudden poloidal phase shift \( \Delta \theta = \pi \) (see for example ref. [64]). Such a phase shift is not observed experimentally [see Fig. 1-a] and expanded view in Fig. 16]. On the other hand, higher density and pressure in the crescent island would tend to stabilize the sawtooth [56]; Park et al. showed numerically that a sufficiently large pressure concentrated on the crescent island side of a 1/1 post-crash resistive kink could nonlinearly stabilize the configuration. Therefore, pressure-based snake models suffer from the fundamental inconsistency that, if the snake density is also the region of higher pressure, either it sits on the wrong side of the magnetic structure to survive sawtooth crashes, or it acts to prevent them. In the later C-Mod snake, such as in Figs. 12, 13, 15 and 16, \( T_e \) observations show that the high density and high temperature fall on opposite sides of the core/crescent division, i.e. the temperature is depressed in the region where the snake density is enhanced. The higher pressure of the higher \( T_e \) circular core drives a partial sawtooth crash, but does not greatly affect the snake density contained in the crescent side. Also, the excess ion density never switches sides of the helical structure as shown in Fig. 16, but maintains a completely regular toroidal rotation transit.
5. Novel 3D MHD model

To address the limitations discussed above, nonlinear resistive MHD simulations have been carried out using compressible 3D MHD equations with density evolution, for toroidal configurations with full plasma shaping and realistic toroidal rotation. The functional form describing the initial helical density perturbation induced by the injection of a fueling pellet or a high-Z impurity has the form $\delta n = (\delta n_0/n_0) f(\psi)H(\theta, \phi)$ where, $H = (1 + \cos(\theta \pm \phi))/2$ [63]; $\theta$ and $\phi$ are the poloidal and toroidal angle, respectively. The function $f(\psi) = \exp \left\{ -(\psi - \psi_1^*)/\Delta \psi \right\} \tanh(2\psi/\psi_1^*)$ is centered at a poloidal magnetic flux surface $\psi_1^*$ near the $q_1$ surface $\psi_1$. For this paper we will discuss the case of a shaped plasma with a small $q_1 < 1$ region, a monotonic safety factor, a helical perturbation centered just inside $q = 1$, a peak magnitude of few percent and coefficients $\psi_1^* = 0.1$, ($r_1^* = 0.2$) and $\Delta \psi = 0.1$.

Nonlinear MHD simulations of the early snake in C-Mod-like plasmas using separate temperature and density evolution, suggest that a 1/1 helical density perturbation localized near $q = 1$ is compatible with the observed early snake formation and also with ongoing sawtooth crashes [63]. For a C-Mod shaped plasma with a small $q_1 < 1$ region and small background toroidal rotation similar to the snake plasmas, the results from the extended M3D initial value code [65, 66] show that a positive helical density of a few percent, in an annulus peaked around or just outside the $q = 1$ radius, drives a new type of internal kink-like mode inside $q < 1$ [see Fig. 17-a]], in such a fashion as to minimize the perturbed 1/1 pressure gradient and free energy. In a poloidal cross section at constant $\phi$, an $m = 1$ perturbation has constant-value contours shaped like back-to-back D’s; $q = 1$ falls at the outer edge of the closely nested “D” contours. Over $q < 1$, the central perturbed pressure and poloidal magnetic flux are closely aligned and resemble a classical 1/1 internal kink mode [see Fig. 17-a] such as $\tilde{\psi}/\tilde{\rho} \simeq (1 - 1/q) \cdot RB/p_0''$. The mode over $q < 1$ is not a linearized $n = 1$ kink, but driven by finite helical density; unlike the classical kink, the density perturbation over $q < 1$ is small so $\tilde{\rho} \simeq n\tilde{T}$. The contours of $\tilde{\psi}$ extending outside $q = 1$ represent the $m = 2$ and 3 poloidal harmonics.

The total density perturbation as shown separately in Fig. 17-b) has a broad helically kinked shape extending beyond $q = 1$ that resembles the early, stage 2 of the C-Mod snake formation. The snake helical density has two components: one piece is peaked on $q \simeq 1$ and extends well outside; the second, inside $q < 1$, is slightly modified by the mode there, as shown also in the composite Fig. 18-a). An important result of the simulation is that a moderate helical density outside a small $q \simeq 1$ radius can be nonlinearly stable and confined on the essentially diffusive time scales of the $q < 1$ mode. The density forms therefore, a long-lived quasi-steady structure that is closely tied to the slowly growing 1/1 mode inside $q < 1$. Over $q \simeq 1$, the temperature tends to be reduced where the perturbed density is large [see Fig. 18-b)]. The $q < 1$ mode grows in such a fashion that $\tilde{T}$ tends to minimize the 1/1 perturbed pressure gradient $\nabla \tilde{p} = \nabla(\tilde{nT} + n\tilde{T})$ in the momentum equation.

The pressure driven 1/1 kink inside $q < 1$ moves towards positive $\tilde{\rho}$ or $\tilde{\psi}$,
Figure 17. (Color online) Nonlinear MHD simulation shows a slowly growing 1/1 kink-type mode over \( q \lesssim 1 \) coupled to a helical density component peaked at or just outside the \( q = 1 \) radius. a) Contours show a poloidal magnetic flux \( \psi \) typical of a 1/1 internal kink instability; \( q = 1 \) falls at the outer edge of the closely nested “D” contours. b) Non-axisymmetric part of \( \delta n \) shown by red(+) / blue(-) shading. Contours extending outside \( q = 1 \) represent the \( m = 2 \) and 3 poloidal harmonics.

Figure 18. (Color online) Composition of non-axisymmetric part of \( \delta n \) shown by red(+) / blue(-) shading with, a) contours of the poloidal magnetic flux \( \psi \) typical of a 1/1 internal kink instability and b) contours of non-axisymmetric part of \( \delta T \).

horizontally to the right (low-field-side) in Fig. 17-a) (see black arrow). The sawtooth crash it produces will redistribute \( p, T, \) and \( n \) over \( q < 1 \) and thus would have relatively little effect on the peak snake density located at \( q = 1 \). Since in the C-Mod ohmic plasma the \( q = 1 \) radius initially grows steadily and axisymmetrically due to the resistive current equilibration, this type of configuration could also be consistent with the gradual transition of the 1/1 density from \( q \simeq 1 \) into a crescent-shaped concentration trapped in a 1/1 island, e.g., from Fig. 10 to 12. For a larger and more unstable \( q < 1 \) region, a similar helical density perturbation can trigger a sawtooth crash [63], seen to initiate some other impurity snakes [4].
6. Conclusions

Snakes are frequently observed in Alcator C-Mod, even in the absence of injection of frozen deuterium pellets, triggered after the central accumulation of high-Z impurities released from the tiles covering the vacuum vessel. Detailed properties of impurity-ion snakes have been described for the first time using a suite of spectroscopic diagnostics that includes AXUV detectors, SXR tomography, a high resolution ECE radiometer and a newly developed high-resolution x-ray imaging spectrometer. Unlike most previous snake measurements, the SXR and AXUV signals can be directly related to the density of molybdenum ion charge states.

The results allow the characterization of the \( n = 1 \) helical structure for quantities such as \( P_{\text{rad}}, n_{\text{Mo}}, n_{e}, T_{e}, Z_{\text{eff}}, \) and \( \eta \) inside the \( q \leq 1 \) region with adequate temporal and spatial resolution. The results show that C-Mod molybdenum snakes that form in the early stage of ohmic discharges begin as a gradually growing helical asymmetry in the impurity ion density. The snake density then typically undergoes a seamless transition from the initial broad kink-like displacement to a crescent-shaped helical structure that fills the \( q < 1 \) region and resembles the magnetic island produced during a resistive internal kink. The smaller circular low density core inside the helical snake concentration undergoes regular sawtooth crashes that temporarily distort the surrounding helical snake density but do not destroy it. The details of the snake evolution and accompanying sawtooth oscillations confirm important differences between the density and temperature dynamics, that rule out explanation by purely pressure-driven processes. Instead, the many of the observed differences can be reproduced by nonlinear 3D MHD simulations that evolve the plasma density and temperature separately, without the need to invoke non-MHD or kinetic processes.

Many questions regarding snake formation, long-term survival, and interaction with other instabilities remain. In C-Mod, snakes also occur under different conditions, including during the flat-top of ohmic discharges and at H\( \rightarrow \)L-mode back-transitions. Future experiments are planned to find reproducible conditions for the snake, including the effects of lower-hybrid heating and current drive (LHCD) and ion-cyclotron resonance (ICRH) RF heating. For example, transient bursts of kink-like oscillations which resemble a fishbone instability are observed when LH current drive is used [67]. Related RF heating and fast-ion effects may be important in fusion burning plasmas such as ITER [68].

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8. Appendix: Use of the Modified Rutherford Equation (MRE)

The Modified Rutherford Equation (MRE) used for describing \((m > 1)\) neoclassical tearing modes (NTMs) in high-\(\beta\) scenarios have been sought to provide a convenient framework to study the formation of the pellet-induced snakes and the destabilizing effect given by an impurity density perturbation in the center of an island (see [3, 9, 11] and references therein). In the absence of neoclassical driving and stabilizing terms in low \(\beta\) plasmas - after the recovery from the initial core thermal perturbation due to the impurity injection, the time evolution of the island width is assumed to be described by,

\[
\frac{\tau_R d\omega}{r_s dt} \approx r_s \Delta' + C_1 \bar{\omega} + C_2 \bar{\omega} + \ldots
\]

(1)

where, \(\bar{\omega} = \omega/r_s\) and \(t^* = t/\tau_R\) are a normalized island size and time; \(r_s\) and \(\Delta'\) are the location of the rational surface and the classical tearing stability term [44]. The second \(\left(C_1 = 3(r_s^2/s)\tilde{P}_{\text{rad}}/\chi_{\text{island}}^4 n_e \langle T_e \rangle\right)\) and third \(\left(C_2 \approx 5.43 g[s(r_s)]\bar{Z}_{\text{eff}}/Z_{\text{eff}}\right)\) terms in equation (1) are proportional to the perturbed radiated power density and \(Z_{\text{eff}}\) carried by the tearing mode [44]. Additional terms in the MRE are also believed to provide additional stabilization. For pellet-induced snakes one can also consider that the enhanced pressure of the snake will provide a bootstrap-like (pressure-driven) current inside the island that will lead to a saturated self-consistent mode. For this case, a correction has been introduced by Smolyakov et al. [11, 69, 70] which is not important for impurity snakes since the density and pressure perturbation is small.

A first glimpse on the importance of each of these terms inside the extended MRE can be assessed using the EFIT-constrained \(q\)-profiles for the times of interest shown in Fig. 16-a); the EFIT-based magnetic shear and the function \(g(s)\) are depicted for completion in Fig. 16-b). With a representative shear \(s \sim 0.2\) at \(R \sim 0.73\) m \((r_s = 0.06\) m) and a core electron thermal diffusivity and temperature of the order of 0.5 m\(^2\)/s and 1.5 keV, the term \(C_1 \approx 4\) at early times and is reduced to \(C_1 \approx 1\) as the radiated power density decreases in time. On the other hand, the function \(g(s) \approx 10\), while the experimentally inferred \(\bar{Z}_{\text{eff}}/Z_{\text{eff}}\) is as high as 70% right after the island is formed and 20% several energy confinement times later. With these data we can safely conclude that the value of the constant \(C_2\) is also reduced in time from nearly 40 to 10, which is an order of magnitude bigger than \(C_1\). Considering a normalized island size near unity, we obtain island saturation \((d\omega/dt \approx 0)\) for anomalously large values of \(-40 < r_s\Delta' < -10\). For comparison, in studying non-linear tearing mode phenomena in tokamaks it is assumed
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Shear profile: \( s(R) = (r/q) dq/dr \)

without justification that \( \Delta' = -2m \) (the vacuum value) for \( m > 2 \) and \( \Delta' = -m \) for \( m = 2 \). The values obtain for the C-Mod impurity-ion snake are similar to the ones estimated by Pecquet, et. al. [9] for Tore-Supra.

References

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