Instability-driven electromagnetic fields in coronal plasmas


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Instability-driven electromagnetic fields in coronal plasmas

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Filamentary electromagnetic fields previously observed in the coronae of laser-driven spherical targets [F. H. Séguin et al., Phys. Plasma. 19, 012701 (2012)] have been further investigated in laser-irradiated plastic foils. Face-on proton-radiography provides an axial view of these filaments and show coherent cellular structure regardless of initial foil-surface conditions. The observed cellular fields are shown to have a constant scale size of \(~210\, \mu\text{m}\) throughout the plasma evolution. A discussion of possible field-generation mechanisms is provided and it is demonstrated that the likely source of the cellular field structure is the magnetothermal instability. Using predicted temperature and density profiles, the fastest growing mode of this instability was found to be approximately constant in time and consistent with the observed cellular size.

Keywords: instability, magnetic field, high-energy-density, proton radiography

PACS numbers: 52.30.-q, 52.35.-g, 52.38.Fz, 52.50.Jm

I. INTRODUCTION

Spontaneous electromagnetic fields can be important to the dynamic evolution of a plasma by directing heat flow \(^1\) as well as providing additional pressures on the conducting fluids through the Lorentz force. Electromagnetic fields are predicted to affect fluid behavior during the core-collapse of supernovae \(^2\) through generation of fields due to hydrodynamic instabilities. In the coronae of stars, self-generated magnetic fields lead to filamentary structure \(^3\) in the hot plasma. Recent experiments by Gregori et al. \(^4\) investigated sources of protogalactic magnetic fields generated by laser-produced shock waves. In inertial confinement fusion (ICF) \(^5\) experiments, self-generated electromagnetic fields can also play a role. Present day laser facilities provide a unique opportunity to study spontaneous field-generation in these extreme environments under controlled conditions.

Self-generated magnetic fields in laser-produced plasmas were first measured by Stamper et al. \(^6\) and were shown to originate from the so-called Biermann battery \(^7\) source caused by perpendicular temperature and density gradients. The magnetic field evolution equation is found by substituting the electron momentum equation into Faraday’s law and can be expressed

\[
\frac{\partial \mathbf{B}}{\partial t} \approx \frac{\nabla T_e \times \nabla n_e}{en_e} + \nabla \times (\mathbf{V}_{\text{adv}} \times \mathbf{B}) - \nabla \times (D_m \nabla \times \mathbf{B}),
\]

where \(T_e\) is the electron temperature, \(n_e\) is the electron density, \(e\) is the unit charge, and \(D_m\) is the diffusion coefficient. In this formalism electron inertia and second order terms in \(\mathbf{B}\) have been neglected. The advection velocity \(\mathbf{V}_{\text{adv}}\) is the vector sum of the fluid velocity \(\mathbf{V}_{\text{fluid}}\) and the so-called Nernst velocity \(\mathbf{V}_{\text{Nernst}}\). Advection by the Nernst effect arises because the magnetic field can move with the heat-conducting electron population and is thus proportional to the temperature gradient

\[
\mathbf{V}_{\text{Nernst}} \approx \frac{\beta_0' \tau_{ei}}{\delta_0 m_e} \nabla T_e ,
\]

where \(m_e\) is the electron mass, \(\tau_{ei}\) is the electron-ion collision time, and \(\beta_0'\) and \(\delta_0\) are Braginskii coefficients. \(^1\) The weakly magnetized approximation (Hall parameter \(\chi << 1\)) has been implemented and for the plastic (CH) plasmas discussed herein \(\beta_0' / \delta_0 \approx 2.5\). The Nernst effect contributes to the total convection of the magnetic field along with field diffusion described by the third term in Equation 1. The first term in Equation 1, the so-called Biermann battery or thermoelectric \(^9\) source, is the dominant generation mechanism of self-generated magnetic fields in plasmas.

Much work has been done in diagnosing spontaneous electromagnetic fields in myriad laser-produced-plasma configurations: irradiated wires \(^11\), directly-driven glass and plastic capsules \(^10,12-14\) inside of hohlraums, \(^15\) and plasma bubble evolution. \(^16,17\) The work described here extends previous observations \(^10\) of filamentary field structures found in the corona of laser-irradiated spherical targets as discussed in Section II. The complexities inherent to the 3-D geometry previously investigated are alleviated through simpler face-on imaging of laser-irradiated plastic foils. The experimental methodology using both monoenergetic-protons and x-rays is discussed in Section III. Radiographic results of coherent 3-D cellular structure in proton images are analyzed and presented in Section IV. This is followed by a discussion of multiple field-generation mechanisms given in Section V and

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it is demonstrated that the likely source of these fields is the magnetothermal instability (MTI) which occurs in the underdense corona. Coronal plasma conditions are shown in Section VI and details of the MTI in these plasmas are discussed in Section VII. This article concludes with a summary of the results presented here and discusses the future directions of this work in Section VIII.

II. FILAMENTARY FIELDS IN THE CORONA

Monoenergetic-proton radiography has been used to image coronal field structures in laser-irradiated CH spheres. Figure 1a shows the experimental setup used in these experiments. A solid CH sphere with a diameter of ~860 µm was irradiated by 0.351 µm laser light with an intensity of ~2×10^{14} W/cm^2 in a 1 ns pulse that had full beam smoothing and SG4 distributed phase plates implemented on the OMEGA laser. The resultant plasma blow-off was probed using 15 MeV fusion-protons provided by the backlighter capsule, discussed further in Section III, and images were recorded on CR-39 nuclear track detectors.

Figure 1b shows four proton-fluence radiographs taken before, during, and after the 1 ns laser pulse. The field of view of the detector is 3 mm×3 mm in each image. Protons are stopped in the solid sphere which results in the white ‘shadow’ seen in each image. No field structures were observed until the end (~1 ns) of the laser drive in these images. Séguin et al. demonstrated that the observed filamentary structures are quickly (~200 ps) generated between ~0.6 ns and ~0.8 ns in these experiments and the amplitude of proton fluence modulations was approximately constant once fields appeared. The filaments expand radially outwards with the blow-off plasma and were still visible after the drive ended.

The magnetic field evolution described by Equation 1 indicates that fields will be advected in the direction of \( \mathbf{V}_{adv} \). The advection velocity depends on both the fluid and Nernst velocities, and changes as a function of position. Plasma evolution in these experiments was modeled using the 1-D radiation-hydrodynamic code LILAC and the resultant velocity profiles at 1 ns are shown in Figure 1c as a function of distance from the ablation surface. The Nernst velocity was calculated using the temperature profiles in Equation 2 and it is predicted to change direction at the peak temperature as indicated in the plot. The advection velocity changes direction in this plasma ~100 µm from the ablation front. B fields generated inside this transition region advect towards the ablation surface, otherwise they expand out with the plasma.

Filamentary structures are observed throughout the coronal plasma, thus fields must be generated in a region where they will advect out with the expanding plasma.
In these images the filaments appear to extend to the ablation surface of the sphere, however the inherent 3-D nature of the spherical geometry precludes any definitive assessment. These images clearly demonstrate that filaments expand with the coronal plasma and are present after the laser drive has ended. The field diffusion time, $\tau_{\text{diff}} \approx (k^2 D_m)^{-1}$ where $k$ is the wave number, may be estimated for $\sim 100$ $\mu$m size fields and is $\gtrsim 5$ ns at the end of the laser drive. Thus it is not surprising that fields are still visible $\sim 500$ ps after the drive ends. Quantitative analysis of the filament size is difficult in this geometry, but was predicted to be of order $\sim 150$ $\mu$m at the quarter-critical surface using Monte Carlo simulations; no difference in size was discernible at different sample times from these data. Face-on imaging of laser-irradiated planar foils has been performed to further probe these filamentary fields on-axis and more accurately quantify their size as described below.

III. PLANAR RADIOGRAPHY EXPERIMENTS

Monoenergetic-proton and x-ray radiography experiments were performed on the OMEGA laser using the configuration shown in Figure 2. Protons are sensitive to both mass and field modulations through Coulomb scattering and the Lorentz force, respectively. X rays are sensitive only to density modulations in the target. The complementarity of these two diagnostic techniques provides information to address density and field distributions during plasma evolution. Unlike the solid sphere experiments, planar foils will be accelerated and Rayleigh-Taylor (RT) growth of density perturbations is expected to occur. Density distributions and growth of perturbations are characterized by x-ray radiographs, whereas protons sample the path-integrated field structures.

The proton backlighter capsules, filled with 18 atm $D^3$He gas, were imploded using 20 OMEGA beams to produce fusion protons. Each proton radiography experiment gives a single ‘snapshot’ in time of the laser-foil interaction and multiple experiments with different laser timings provide a series of radiographs illustrating the plasma evolution. This backlighting technique provides a temporal resolution of $\sim 150$ ps and $D^3$He-fusion protons ($E_p \sim 15$ MeV) are produced by an approximately Gaussian source with a FWHM of $\sim 45$ $\mu$m. 15-MeV proton radiographs were recorded on filtered CR-39 plastic nuclear track detectors. After exposure, CR-39 samples were etched to reveal tracks left by the incident protons. Through the etching process, signal tracks in the plastic were revealed and pieces were scanned using an optical microscope system. From these scans, radiographs were processed and proton fluence images were created.

X-ray radiographs were taken using a laser-irradiated Uranium foil to generate $\sim 1.3$ keV x rays for optimum contrast through $\sim 20$ $\mu$m CH. Images were recorded on film using a framing camera with a temporal resolution of $\sim 80$ ps and a spatial resolution of $\sim 10$ $\mu$m. The apertured framing camera provided multiple images of a single foil during its evolution, yielding multiple radiographs from a single experiment. The measured optical depth image may be directly converted to an areal density map of the target for comparison with proton radiographs that are sensitive to both density modulations and field deflections at the foil.

Four types of CH foils of varying thicknesses and surface perturbations were used in these experiments and are listed in Table I. Foil surfaces were either flat, seeded with ridge-like 2-D sinusoidal modulations, or 3-D eggcrate-like sinusoidal modulations. The laser spot was shaped by SG4 distributed phase plates (DPP) smoothed by spectral dispersion (SSD), and distributed polarization rotators (DPR) implemented to minimize the broadband imprint from the laser spot. Twelve beams were overlapped to provide a drive intensity of $I \lesssim 4 \times 10^{14}$ W/cm² within a $\sim 750$ $\mu$m diameter spot using a 2 ns square pulse to deliver a total of $\sim 3300$ J of

### Table I

<table>
<thead>
<tr>
<th>Label</th>
<th>$l_0$ [µm]</th>
<th>$\lambda_0$ [µm]</th>
<th>$a_0$ [µm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat Foil</td>
<td>21</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>120 µm (2-D)</td>
<td>21</td>
<td>120</td>
<td>0.27</td>
</tr>
<tr>
<td>180 µm (2-D)</td>
<td>23</td>
<td>180</td>
<td>0.55</td>
</tr>
<tr>
<td>115 µm (3-D)</td>
<td>26</td>
<td>115</td>
<td>0.56</td>
</tr>
</tbody>
</table>

FIG. 2. A schematic drawing of the experimental setup used to radiograph directly-driven plastic foils using (a) protons and (b) x rays. Proton and x-ray images were recorded on CR-39 and film, respectively.
energy on target. Proton and x-ray radiographs were taken at times up to 2.4 ns to provide data during and after the onset of the laser drive.

IV. EXPERIMENTAL RESULTS

A. Proton Radiographs

A comprehensive summary of proton-fluence radiographs is shown in Figure 3. Radiographs were taken of the four different foil types after the first nanosecond of the 2 ns laser drive. Proton fluence images of all foil types for times \( \lesssim 1.5 \) ns indicate minor variance across the analysis region, though coherent linear features were observed in 2-D modulated foils. Previous work \(^{29,30}\) has shown that these linear features are caused by Rayleigh-Taylor-induced magnetic fields caused by the growth of the preimposed surface perturbations. A rapid transition (\( \lesssim 200 \) ps) occurs near \( t \sim 1.5 \) ns whereby the fluence radiographs of all foil types show a drastic change in appearance. Some underlying linear features most likely due to RT-induced fields are still observable, especially 180-µm-foil images at times \( \gtrsim 1.6 \) ns, but the 3-D cellular structure is prevalent and dominates proton radiographs. These features are consistent with an axial view of the previously observed filamentary fields.

Cellular structure was shown to begin during the laser drive and continued well after the end of the pulse. The dominant scale size of these features (\( \lambda_{AC} \)) and the rms amplitude modulation (\( \sigma_{rms} \)) were calculated to characterize the scale and strength of the filamentary field structures. The results of the autocorrelation analysis (as described in the Appendix) are plotted in Figure 4a. It is clearly demonstrated that these features have an approximately constant scale size that can be characterized by \( \lambda_{AC} \approx 210 \) µm with a standard deviation of \( \pm 30 \) µm. Radiographs taken before cellular-onset were analyzed, but did not reveal a dominant scale size and are thus not shown in Figure 4a. The scale of filamentary fields is shown to be constant in size immediately after initial on-
set and it does not change in time or depend on the initial foil surface conditions. This suggests that filamentary fields are generated away from regions affected by the shape of the ablation surface, i.e., in the expanding, underdense corona.

Figure 4b illustrates the normalized rms amplitude as a function of time for all four foil types. Radiographs at earlier times ($\lesssim 1.5$ ns) are shown to have normalized rms amplitudes of $\lesssim 20\%$, though no dominant scale size was observed. The normalized broadband rms characterizes the amplitude of proton deflections and thus path-integrated field strength. It is clearly shown that proton deflections grow at the same rate for all foil types during the laser drive. The filamentary fields causing the cellular features are created during the laser pulse, but do not lose appreciable strength after the drive ends.

B. X-ray Radiographs

All four foil types were radiographed with $x$ rays to characterize density distributions in these laser-foil interactions. Modulations in areal density arise due to Rayleigh-Taylor growth of surface perturbations and laser imprint. X-ray radiographs shown in Figure 5a illustrate the evolution of areal density modulations for all target types. In these images, lighter pixels indicate higher areal density in the target (more $x$-ray absorption). Flat-foil radiographs show no significant features until late in time ($t \sim 2.2$ ns), at which point small-scale ($\sim 30$ $\mu$m) structure due to laser imprint becomes apparent. However, $x$-ray radiographs of modulated foils clearly show dramatic features at the seeded wavelengths due to RT growth. The perturbation amplitude of the $\lambda = 120$-$\mu$m-foil increases in time with a rate of $\gamma_{120} \sim 2$ ns$^{-1}$ during the drive and these data agree well with DRACO$^{33}$ radiation-hydrodynamic simulations as shown in Figure 5b. Some small-scale structure, also at $\sim 30$ $\mu$m, is visible at late times in modulated foils, but these features are much lower in amplitude than the dominant preimposed modulations.

X-ray radiographs taken at times $\gtrsim 1.5$ ns do not show similar features as those observed in late-time proton fluence images. Cellular structure in areal density has been observed$^{31}$ under different experimental conditions due to laser imprint, but in experiments discussed here the 3-D structure did not have enough time to strongly de-

FIG. 5. (a) Summary of sample x-ray radiographs of the four different foil types. Images were taken during and after the laser drive as indicated by the image location relative to the pulse schematic. Those foils with preimposed surface modulations demonstrate RT-growth of the seeded perturbations. (b) Analysis of the 120 $\mu$m data show good agreement with radiation-hydrodynamic predictions of RT-growth.
A Fourier analysis was performed on the x-ray images of flat and 120-μm-modulated foils at ∼2.2 ns to compare the relative amplitudes of the observed features. The resultant spectra are shown in Figure 6 and demonstrate that the ∼30 μm features in both cases are approximately equal in amplitude and are consistent with RT growth of laser-imprinted perturbations. In the modulated foil case, the amplitude of 120 μm perturbations is ∼5 times higher than ∼30 μm features.

Proton deflections due to RT-induced B fields are dominated by fields at the seeded wavelength of 120 μm in the modulated foil case at times ≲1.5 ns. B fields created by RT growth occur near the ablation surface and, neglecting diffusion, are proportional to the fluid vorticity. The peak field scales with perturbation parameters as |B(t)| ∝ h(t)/γ, where h is the perturbation amplitude, λ is the wavelength, and γ is the growth rate. Proton deflections, though, are proportional to the path-integrated field strength ⟨BL⟩ where the field scale length L_B ∼ h. Measured RT growth rates for both wavelengths, γ_30 ∼ 4.5 ns⁻¹ and γ_120 ∼ 2 ns⁻¹, were used with the fact that h ∝ ⟨pL⟩_{rms} to estimate the relative magnitudes of ⟨BL⟩ between the two perturbation wavelengths. If proton deflections late in time were due only to RT-induced B fields, simple estimates show that deflections due to λ=120 μm are ∼15 times higher than those for λ=30 μm. This is a lower limit because diffusion effects, not included here, affect shorter wavelengths more than longer ones. Furthermore, this analysis is for images at ∼2.2 ns, whereas proton images illustrate strong cellular features by ∼1.7 ns. If RT-induced B fields were the dominant field structure in modulated-foil experiments, proton images would exhibit strong features consistent with surface perturbations. Instead, coherent cellular structures likely caused by filamentary fields in the underdense corona were observed in proton images at late times independent of initial surface conditions.

V. INSTABILITIES IN LASER-PRODUCED PLASMAS

Magnetic fields may be generated by a range of instabilities and sources as described in detail by Haines.9 The primary sources are outlined in Figure 7a that shows where they occur in the sample plasma given by DRACO profiles at 1.5 ns. In addition to these plasma instabilities, laser-plasma instabilities (LPI) can locally generate B fields through generation of hot-electron currents in the coronal plasma. Hard x-ray detectors35 and scattered-light streak cameras36 were fielded to observe LPI-related plasma behavior in the corona, though no hard x-ray or relevant scattered-light signals were observed. Filamentation of laser ‘hot-spots’ was not responsive because the average intensity was too low37 and all beams were spectrally smoothed29. LPI-generated hot-electron currents are not the source of these coherent field structures.

The so-called Weibel instability is generated by electron-temperature anisotropy in the plasma. The typical collisionless form (CLW) of this instability is only relevant at the very edge of the corona when λ_{mfp}/c/ω_{pe} ≳ 10³, where c/ω_{pe} is the collisionless skin depth. In these experiments, this parameter is ≲ 500 and thus CLW is a very unlikely field source. The collisional Weibel (CW) instability, which occurs in the denser plasma regions, has been predicted to generate fields under ablatively driven conditions.39 Fields created by the CW tend to grow fastest near the over dense region (n_e ≳ n_{crit}), but when the Nernst effect and field diffusion were included, the instability was shown to be stabilized. Nonetheless, if these fields exist under the specific conditions.
VI. PLASMA FLOW CONDITIONS

Laser-solid interactions create plasma conditions that vary as a function of position from the solid material as demonstrated in Figure 7a. Plasma conditions go from cold \((T_e \sim 200 \text{ eV})\) and dense \((n_e \sim 10^{23} \text{ cm}^{-3})\) near the ablation front to hot \((T_e \sim 2 \text{ keV})\) and sparse \((n_e \sim 10^{21} \text{ cm}^{-3})\) in the corona. The fluid velocity also varies and, at the time shown in Figure 7b, changes direction from inward flow to outward expansion just outside the critical surface. These varied conditions give rise to differences in dominant physics mechanisms.

The magnetic Reynolds number \((R_{em})\) relates B-field advection to diffusion in the plasma. This can be expressed as \(R_{em} = V_{adv}L/D_m\), where \(V_{adv}\) is the field advection velocity, \(L\) is the plasma scale length, and \(D_m\) is the field-diffusion coefficient. Figure 8 shows how \(R_{em}\) varies in the plasma described in Figure 7a and demonstrates that in most locations, field advection dominates over diffusion. This is due to the low resistivity away from the ablation front and is consistent with B fields being frozen-in to the flow.

The Reynolds number \((Re)\) characterizes whether the fluid flow is laminar \((Re \lesssim 2300)\) or turbulent \((Re \gtrsim 4000)\). This dimensionless number compares inertial to viscous forces and can be written \(Re = V_{\text{fluid}}L/\nu\), where \(V_{\text{fluid}}\) is the fluid velocity, \(L\) is the plasma scale length, and \(\nu\) is the kinematic viscosity of plasma ions. Figure 8 illustrates how \(Re\) changes as a function of position. The Reynolds number increases near the ablation front due to the decrease in viscosity \((\nu)\), though x-ray radiographs demonstrated coherent RT-growth at the ablation front in modulated foils during the entirety of the laser drive. These calculations demonstrate that plasma flow is dominantly laminar in nature allowing for coherent features and that B fields will be strongly advected in the underdense corona.

VII. MTI IN SPHERICAL AND PLANAR GEOMETRIES

The MTI grows from a seed magnetic field in the coronal plasma that can originate from a number of different perturbative sources, laser nonuniformities, plasma waves, etc. Thus, the initial mode distribution is not well understood or characterized and varies spatially and temporally, making quantitative analysis of this instability very difficult. Some insight may be gained,
however, by examining the growth rate ($\gamma_{MTI}$) and wavelength($\lambda_{MTI}$) of the fastest growing mode in these CH plasmas given by\textsuperscript{42}

$$\gamma_{MTI} \approx 1.65 \times 10^8 \frac{T_e^{5/2}}{n_e Z_L n_T \ln \Lambda} \left[ \frac{1}{\text{ns}} \right],$$

$$\lambda_{MTI} \approx 2 \times 10^{-4} \sqrt{\frac{L_T Z}{n_e \lambda_0^3 T_e^{1/2}}} \left[ \mu\text{m} \right].$$

In the preceding equations $Z$ is the average charge state, $\ln \Lambda$ is the Coulomb logarithm, the electron temperature $T_e$ is in keV, electron density $n_e$ is in $10^{20}$ cm$^{-3}$, the Debye length $\lambda_0$ is in $\mu$m, and the scale lengths for temperature ($L_T$) and density ($L_n$) are also in $\mu$m. Figure 9a and b illustrate calculations from LILAC and DRACO profiles, respectively, of $\lambda_{MTI}$ (solid) and $\gamma_{MTI}$ (short dash) at single instances near the observed field-onset time. The fastest growing wavelength is very large near the peak temperature due to the long temperature scale lengths, but levels off quickly and slowly increases farther out due to decreasing densities in both the spherical and planar cases. The growth rate peaks at $\sim 5$ ns$^{-1}$ in the planar case and decreases thereafter, whereas $\gamma_{MTI}$ levels off at $\sim 10$ ns$^{-1}$ in the spherical case before diverging at larger radii due to the rapidly decreasing density. These profiles change as a function of time, but the snap-shots shown here provide the general differences between the planar and spherical cases.

The observed onset time of the coherent field structures in planar experiments was $\sim 2$ times later than the apparent onset in spherical experiments. The exact cause of the apparently rapid onset of these fields is not well understood. Though near the field-onset time, calculations from classic MTI theory, as shown in Figure 9a and b, predict a growth rate $\sim 2$ times faster in spherical experiments than planar experiments, consistent with the two separate data sets. A given plasma profile provides a spectrum of ‘fastest growing modes’ throughout the plasma, though in both the spherical and planar cases these modes are predicted to be between $\sim 200$-300 $\mu$m, again consistent with observations. To assess some aspect of temporal variance, an average wavelength $<\lambda_{MTI}>$ is defined by

$$<\lambda_{MTI}> = \frac{\sum_i \lambda_{MTI,i} \times \gamma_{MTI,i}}{\sum_i \gamma_{MTI,i}},$$

where each $\lambda_{MTI,i}$ is weighted by the associated growth rate and is summed over the MTI-unstable region in the underdense corona at each time step. The results of this calculation for both spherical and planar simulations are shown in Figure 9c. The laser drives for both configurations are also shown (dotted) for reference and it is clear that $<\lambda_{MTI}>$ rapidly increases after the drive turns off due to flattening of the temperature profiles as the plasma cools. These calculations suggest that MTI occurs earlier in the drive at smaller wavelengths, though this was not observed in experiments. Rather, a rapid transition ($\sim 200$ ps) was demonstrated in both the spherical and planar experiments. Interestingly though, the measured characteristic size of cellular structures, $\sim 210$ $\mu$m, crosses the spherical and planar curves in Figure 9c near the observed onset times for each case. It is also important to note that these calculations predict a slowly varying dominant scale size in time, which

\[FIG. 9.\] The fastest growing mode growth rate $\gamma_{MTI}$ (short dash) and wavelength $\lambda_{MTI}$ (solid) profiles are shown near field onset times for (a) the spherical case at 0.8 ns and (b) the planar case at 1.5 ns. Simulated profiles are used with Equations 3 and 4 to generate the curves shown here and plotted as a function of (a) radial distance in and (b) axial distance. An average wavelength $<\lambda_{MTI}>$ was calculated at each time by averaging $\lambda_{MTI}$ over this space and using the growth rate as a weighting factor. (c) The resulting average wavelength as a function of time for the planar (solid) and spherical (dash-dot) cases with the respective drives (dotted) shown at the bottom in arbitrary units. The inferred characteristic size of cellular structures determined from planar proton radiographs is also shown at $<\lambda_{MTI}> = 210$ $\mu$m.

![Diagram showing fastest growing mode growth rate and wavelength profiles near field onset times for spherical and planar cases.](image)

- $\gamma_{MTI}$: Growth rate
- $\lambda_{MTI}$: Wavelength
- $<\lambda_{MTI}>$: Average wavelength
- Spherical: 0.8 ns
- Planar: 1.5 ns
- Drives: LILAC, DRACO
- Characteristics:
  - $<\lambda_{MTI}> = 210$ $\mu$m
- Observations:
  - Fastest growing wavelength near peak temperature
  - Slow increase at larger radii
  - Rapid transition (200 ps)
  - Characteristic size matches measured values.
was not observed. Even though no temporal growth was measured from proton radiographs, the range of values measured from separate planar experiments is consistent with the predicted \(\sim 200-300 \mu m\) during these times.

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Appendix A: Proton-Radiograph Analysis Techniques

The two observables of interest in proton radiographs are the characteristic spatial wavelength of the features ($\lambda_{AC}$), if one exists, and the normalized broadband rms amplitude ($\sigma_{rms}$) which is a measure of the strength of the features. The latter may be calculated from the dis-
ttribution of fluence values after removing the statistical component,

\[ \sigma_{\text{rms}} = \sqrt{\frac{\sigma_{\text{meas}}^2 - \sigma_{\text{stat}}^2}{\mu_{\text{meas}}}} \]  

(A1)

where \( \sigma_{\text{meas}} \) is the measured statistical deviation of protons per pixel, \( \mu_{\text{meas}} \) is the statistical mean proton fluence per pixel used to normalize the variation across different experiments, and \( \sigma_{\text{stat}} \) is the numeric statistical variation per pixel \( \sim \sqrt{\mu_{\text{meas}}} \). Deducing the characteristic spatial length of the nonlinear features requires a more complex analysis.

An autocorrelation (AC) algorithm is used on each proton-fluence image to determine the isotropic scale-size of the observed features. This procedure begins by calculating the 2-D FFT of a square region that results in a 2-D array \( C(k_x, k_y) \) of complex Fourier coefficients,

\[ C(k_x, k_y) = \mathcal{F}\{I(x, y)\} \]  

(A2)

where \( \mathcal{F} \) denotes the FFT algorithm and \( I(x, y) \) is the array of pixel values at the area of interest. The autocorrelation coefficients \( A(x, y) \) may then be calculated directly from the Fourier coefficients,

\[ A(x, y) = \mathcal{F}^{-1}\{C^*C\} \]  

(A3)

where \( C^* \) is the complex conjugate of the Fourier coefficient array and \( \mathcal{F}^{-1} \) is the inverse FFT algorithm. The autocorrelation coefficients, as defined above, represent how well the image correlates with itself. The AC coefficients are azimuthally averaged in space, thus eliminating one of the spatial dimensions and producing a 1-D (radial) representation of the AC coefficients. If present, an isotropic scale-size will be revealed by this 1-D representation and may be extracted from each proton radiograph.

Figure 10 illustrates two examples of the AC analysis with synthetic data and two examples from experimental radiographs. Two synthetic images were generated with 2-D sinusoidal functions (eggcrates) of different amplitudes with a wavelength \( \lambda_0 \) of 150 \( \mu \)m in both directions, resulting in a diagonal peak-to-peak wavelength \( \lambda_{0,D} = (\sqrt{2}/2)\lambda_0 \) of 106 \( \mu \)m. The two images in Figure 10a illustrate the differences between amplitudes of 10% (top) and 30% (bottom) of the mean. The corresponding normalized 1-D AC spectra are shown by the solid and dashed lines, respectively and show that higher amplitudes in the image result in higher amplitudes in the AC coefficients. The synthetic images used have a single spatial frequency over many wavelengths, which results in the decaying oscillations observed at harmonics of the fundamental wavelength (first peak) in the AC spectra. The dominant scale size was measured to be \( 113\pm8 \mu \)m in both cases as illustrated by the dotted line. This measurement is consistent with the original diagonal peak-to-peak wavelength; AC coefficients near the lateral wavelength of 150 \( \mu \)m averaged out during the process. This analysis procedure accurately measures the dominant scale size in these images to within the uncertainty of the measurement.

The dominant scale size of isotropic features in an image was calculated from the 1-D AC spectrum by fitting a curve to the first observed peak at a length greater than zero. A 2nd-degree polynomial was found to fit most data better than a Gaussian, or other peaked functions. The primary goal of the fit is to obtain an accurate measurement of the peak algorithmically. Furthermore, using the fitted curve, an uncertainty in the measured peak position may be estimated. The width of the peak represents the uncertainty in the dominant scale size of the observed features, though the typical FWHM metric is not a well-defined quantity in most cases. Therefore, the width of the parabola is taken at the point where it has reached 95% of the value at the peak. This width is the uncertainty in the measured scale size of cellular features.

Figure 10b illustrates two sample proton radiographs of laser-irradiated CH foils with \( \sim 120 \mu \)m ridge-like perturbations. The first image at \( t\sim 1.4 \) ns shows the expected linear behavior in the image and the corresponding AC coefficient spectrum (solid) does not indicate any peak after the initial fall off, suggesting that there is no dominant, isotropic scale-size. The bottom image in Figure 10b occurs later in time, at \( t\sim 1.6 \) ns, and the corresponding AC coefficient spectrum (dashed) is shown.
This spectrum shows a faster fall off, indicating higher amplitude modulations than the previous image, and a dominant scale size peak at 185±10 µm. This systematic analysis technique provides an accurate measurement of the dominant scale size, when present, of isotropic features observed in radiographic images.