The long-time quasilinear development of the free electron laser instability is investigated for a tenuous electron beam propagating in the z-direction through a helical wiggler field $\mathbf{B}(z) = -\mathbf{B} \cos k_0 z \hat{\mathbf{x}} - \mathbf{B} \sin k_0 z \hat{\mathbf{y}}$. The analysis neglects longitudinal perturbations ($\delta \phi = 0$) and is based on the nonlinear Vlasov-Maxwell equations for the class of beam distributions of the form $f_{b}(z,p_x,t) = n_0 \delta(p_{x}) \delta(p_{y}) \times G(z,p_{z},t)$, assuming $\delta/\delta x = 0 = \delta/\delta y$. The long-time quasilinear evolution of the system is investigated within the context of a simple "water-bag" model in which the average distribution function $G_{0}(p_{z},t) = (2\pi)^{-1} \int_{-\infty}^{\infty} dz G(z,p_{z},t)$ is assumed to have the rectangular form $G_{0}(p_{z},t) = (2\Delta(t))^{-1}$ for $|p_{z} - p_{0}(t)| \leq \Delta(t)$, and $G_{0}(p_{z},t) = 0$ for $|p_{z} - p_{0}(t)| > \Delta(t)$. Making use of the quasilinear kinetic equations, a coupled system of nonlinear equations is derived which describes the self-consistent evolution of the mean electron momentum $p_{0}(t)$, the momentum spread $\Delta(t)$, the amplifying wave spectrum $|H_{k}(t)|^{2}$, and the complex oscillation frequency $\omega_{k}(t) + i\gamma_{k}(t)$. These coupled equations are solved numerically for a wide range of system parameters, assuming that the input power spectrum $P_{k}(t=0)$ is flat and non-zero for a finite range of wavenumber $k$ that overlaps with the region of $k$-space where the initial growth rate satisfies $\gamma_{k}(t=0) > 0$. To summarize the qualitative features of the quasilinear evolution, as the wave spectrum amplifies it is found that there is a concomitant decrease in the mean electron energy $\gamma_{0}(t)m_{e}c^{2} = \left[ m_{e}^{2}c^{4} + e^{2}B^{2}/k_{0} + p_{0}^{2}(t)c^{2} \right]^{1/2}$, an increase in the momentum spread $\Delta(t)$, and a downshift of the growth rate $\gamma_{k}(t)$ to lower $k$-values. After sufficient time has elapsed, the growth rate $\gamma_{k}$ has downshifted sufficiently far in $k$-space that the region where $\gamma_{k} > 0$ no longer overlaps the region where the initial power spectrum $P_{k}(t=0)$ is non-zero. Therefore, the wave spectrum saturates, and $\gamma_{0}(t)$ and $\Delta(t)$ approach their asymptotic values.

---

Permanent address: Plasma Fusion Center, Massachusetts Institute of Technology, Cambridge, Mass., 02139

Permanent address: Institute of Electron Physics, Academia Sinica, Beijing People's Republic of China