A THEORETICAL STUDY OF
COHERENT STRUCTURES
IN NONNEUTRAL PLASMA COLUMNS

by

Steven M. Lund

Submitted to the Department of Physics on August 20, 1992
in partial fulfillment of the requirements for the
Degree of Doctor of Philosophy in Physics

ABSTRACT

A ubiquitous feature of experimental and computer simulation studies of magnetically confined pure electron plasmas in cylindrical confinement devices is the formation of nonaxisymmetric ($\partial/\partial \theta \neq 0$) rotating equilibria. In this dissertation, nonaxisymmetric rotating equilibria are investigated theoretically for strongly magnetized, low-density ($\omega_{pe}^2/\omega_{ce}^2 \ll 1$) pure electron plasmas confined in a two-dimensional cylindrical geometry. These dynamic equilibria are also called rotating coherent structures, and are stationary (time-independent) in a frame of reference rotating with angular velocity $\omega_r = const.$ about the cylinder axis ($r = 0$). Radial confinement of the pure electron plasma is provided by a uniform axial magnetic field $B_0 \hat{e}_z$, and a grounded, perfectly conducting, cylindrical wall is located at radius $r = r_\omega$. The analysis is based on a nonrelativistic, guiding-center model in the cold-fluid limit (the continuity and Poisson equations) that treats the electrons as a massless fluid ($m_e \to 0$) with $E \times B$ flow velocity $V_z = -(c/B_0)\nabla \phi \times \hat{e}_z$. Within this model, general rotating equilibria with electron density $n_e \equiv n_N(r, \theta - \omega_r t)$ and electrostatic potential $\phi \equiv \phi_N(r, \theta - \omega_r t)$ have the property that the electron density is functionally related
to the streamfunction $\psi_n = -e\phi_n + \omega_r (eB_0 / 2c) r^2$ by $n_n = n_n(\psi_n)$. The streamfunction $\psi_n$ satisfies the nonlinear equilibrium equation $\nabla^2 \psi_n = -4\pi e^2 n_n(\psi_n) + 2\omega_r eB_0 / c$ with $\psi_n = \omega_r (eB_0 / 2c) r^2 \equiv \psi_w = const.$ on the cylindrical wall at $r = r_w$. A general methodology for the solution of this equilibrium system is presented and several properties of rotating equilibria are analyzed. Following this analysis, two classes of nonaxisymmetric equilibria are investigated. These two classes of equilibria can have large amplitude (strongly nonaxisymmetric). First, a class of vortex-like rotating equilibria is analyzed that is characterized by a structured density profile that fills a confinement geometry with an inner conducting cylinder at radius $r = r_c < r_w$. The streamfunction describing these vortex-like equilibria is derived exactly and analyzed in several relevant limits. Next, a physically motivated class of rotating equilibria with "waterbag" (step-function) density profiles and free plasma-vacuum interfaces is investigated. An integral equation formulation of the nonlinear equilibrium equation that describes general waterbag equilibria is developed. Then, a numerical method that can be used to construct diverse varieties of solutions for highly nonlinear waterbag equilibria is formulated. This method is employed to examine two classes of nonaxisymmetric equilibria that are nonlinear extrapolations of well-known small-amplitude equilibria. These two classes of rotating equilibria bear strong similarities to coherent structures observed experimentally by Driscoll and Fine [Phys. Fluids B 2, 1359 (1990)].

Thesis Supervisor: Dr. Ronald C. Davidson

Title: Professor of Physics