Experimental Energy Confinement Time Scaling with Dimensionless Parameters in C-Mod I-mode Plasmas

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Abstract

The dependence of energy confinement time on gyroradius, $\beta$, and normalized collisionality in I-mode plasmas is investigated through dimensionless parameter scans in C-Mod. The gyroradius scaling is calculated to be $\Omega \tau_E \propto \rho_*^{-3.9 \pm 1.5}$, which suggests core transport may scale with gyro-Bohm physics, indicating favorable extrapolation of the I-mode regime to future devices at low $\rho_*$. The scaling exponent for $\nu_C$ is calculated to be small, but positive ($\Omega \tau_E \propto \nu_C^{0.44 \pm 0.24}$), and the exponent for $\beta$ is deemed inconclusive ($\Omega \tau_E \propto \beta^{1.4 \pm 3.1}$) due to the low statistical significance in the dataset and therefore requires further investigation. The individual dimensionless parameter scaling is compared to calculations from larger C-Mod I-mode datasets as well as multi-machine scaling laws for ELMy H-mode and L-mode plasmas. Multiple regression techniques are used to analyze parameter significance within the dataset, and the $\rho_*$ variable is shown to dominate the sensitivity of the energy confinement time.

I. Introduction

A great challenge in fusion energy is accurately predicting how physics in current tokamaks will scale to reactor relevant plasma regimes. The energy confinement time, or energy loss per unit volume, is of particular interest because of the role it plays in describing economical steady state power production and ignition as understood by the Lawson criterion [1]. Due to the complex nature of core transport processes and the requirement of known plasma profiles and their gradients to assess accurately a physics based understanding of the plasma energy confinement time, often 0D models developed from databases are employed to extrapolate energy confinement times to future devices. Dimensionally similar plasmas are useful for this extrapolation because physics quantities such as the thermal diffusivity, $\chi$, are assumed to scale upon proper choice of dimensionless parameter. Tokamak experiments today can achieve
matched dimensionless parameters to reactor scale devices with the exception of normalized
gyroradius, $\rho_*$, which provides particular motivation for understanding the scaling with this parameter.

Dimensionless parameter theory [2-4] and database studies [5-11] show that energy
confinement time scaling can be expressed in terms of a power law using dimensional
parameters in the form of Eq. (1).

$$\tau_E \propto I_p^{\alpha_I} B_T^{\alpha_B} \bar{n}_e^{\alpha_{ne}} P_{loss}^{\alpha_{P}} R^{\alpha_R} \kappa^{\alpha_k} \epsilon^{\alpha_{\epsilon}} \quad (1)$$

where $I_p$ is the plasma current, $B_T$ the toroidal magnetic field, $\bar{n}_e$ the line average electron
density, $P_{loss}$ the loss power across the separatrix, $R$ the major radius, $\kappa$ the plasma elongation,
and $\epsilon$ the inverse aspect ratio. The $\alpha$ exponents are derived through experimental fits, with a
good example being the multi-machine ITPA scaling law calculated for standard H-mode and L-
mode plasmas used to extrapolate the energy confinement time to ITER [6-9]. Assuming size
and plasma shape invariance, Equation (1) can be recast through the methodology outlined in
reference [5] to be expressed in terms of dimensionless parameters.

$$\Omega \tau_E \propto \rho^{*\alpha_{\rho}} \beta^{\alpha_{\beta}} \nu_C^{\alpha_{\nu_C}} q^{\alpha_q} \quad (2)$$

$$\rho_* = \frac{\sqrt{m T}}{a B_T}$$

$$\beta = \frac{<\frac{n T}{\rho}>}{(B^2/2\mu_0)}$$

$$\nu_C = \frac{\bar{n}_e R}{<\frac{T_e^2}{\rho}>}$$

where $\Omega$ is the cyclotron frequency, and the normalized confinement time is expressed through
the product $\Omega \tau_E$, and is often simplified to $B \tau_E$. The relevant dimensionless parameters used in
the power law are the gyroradius ($\rho_*$), the ratio of plasma pressure to magnetic pressure ($\beta$), the
volume averaged electron collisionality ($\nu_C$), and the safety factor ($q$). In this study, the
gyroradius is defined with ion quantities, the normalized collisionality with electron quantities,
and the plasma $\beta$ with the toroidal magnetic field pressure. Comparing the confinement times of
plasmas with the same $\beta$, $\nu_c$, and $q$, but varied $\rho$, values allows the exponent $\alpha_{\rho}$ to be calculated (and similarly for $\alpha_{\beta}$, $\alpha_{\nu}$, and $\alpha_{q}$).

Similar dimensional analysis regressions have been employed on conventional regimes like L-modes and H-modes with Edge Localized Modes (ELMs) for extrapolation to future devices like ITER. However, there has been significant effort in developing confinement regimes that are intrinsically stable to ELMs [12-13], which can be detrimental to plasma facing components and reduce the operational lifetime of a steady state tokamak. One of these regimes is the I-mode, which lacks a particle transport barrier, but maintains a thermal transport barrier [14-18]. By leveraging edge pedestal turbulence, the peeling-ballooning stability limits for ELM onset can be avoided through reduction of the pressure gradient in the edge. Access to I-mode is most readily obtained by operating with ion $\nabla B$ drift directed away from the active X-point, which is typically considered the “unfavorable” direction for H-mode access. The window of I-mode operation widens with magnetic field strength [17], making it an interesting prospect as a viable reactor regime, and motivates further understanding of how the energy confinement time scales.

II. Experimental methodology for scanning dimensionless parameters $\rho$, $\nu_c$, and $\beta$

Previous C-Mod experiments performed engineering parameter scans in plasma current, line average density, and toroidal magnetic field to study the confinement time scaling, shown in Fig. 9 of reference 17. These scans found a small to zero dependence on $I_p$ ($\alpha_{I_p} = 0.36$) and $B_T$ (no fit), and a modest positive scaling with $n_e$ ($\alpha_{n_e} = 0.41$), but were comprised of a small number of data points. These dimensional parameter scalings were also investigated in reference 11 to a database of C-Mod I-modes with varied shape, $\nabla B$ drift direction, and divertor configuration to determine Eq. (3).

$$\tau^\text{database}_E = (0.014 \pm 0.002) \times I_p^{0.685 \pm 0.076} B_T^{0.768 \pm 0.072} n_e^{0.017 \pm 0.048} P_{\text{loss}}^{-0.286 \pm 0.042}$$  \hspace{1cm} (3)$$

The exponents from the database [11] differ from the dedicated parameter scans [17]. The database regression calculates a larger dependence of confinement time on plasma current and toroidal magnetic field, as well as negligible dependence a with line average density as compared to a positive dependence seen in the dedicated parameter scans. These discrepancies motivate...
additional analysis and experiments. Specifically, the dimensional database can be transformed into dimensionless parameter space and compared to dedicated experiments.

The power law fit to I-mode energy confinement times in reference 11 can be non-dimensionalized to the form of Eq. (2) by calculating dimensionless exponents using Eqs. (4) [5],

\[
\begin{align*}
\alpha_\rho &= \left[ -\frac{3}{2} \alpha_I - \frac{3}{2} \alpha_B - 3 \alpha_P - 2 \alpha_n \right] / (1 + \alpha_p) - 3/2 \\
\alpha_\beta &= \left[ \frac{1}{4} \alpha_I + \frac{1}{4} \alpha_B + \frac{3}{2} \alpha_P + \alpha_n \right] / (1 + \alpha_p) + 1/4 \\
\alpha_v &= \left[ -\frac{1}{4} \alpha_I - \frac{1}{4} \alpha_B - \frac{1}{2} \alpha_P \right] / (1 + \alpha_p) - 1/4 \\
\alpha_q &= -\alpha_I / (1 + \alpha_p)
\end{align*}
\]

to yield the scaling in Eq. (5).

\[
\Omega_T^{database} \propto \rho_*^{-3.7 \pm 0.2} \beta^{0.59 \pm 0.24} v_c^{-0.61 \pm 0.03} q^{-0.72 \pm 0.15}
\]

The uncertainties in the dimensionless parameter exponents are calculated by applying the transformation in Eqs. (4) to the full range of possible exponents including the uncertainties in Eq. (3). It is notable that the exponents in this estimated I-mode power law suggest stronger than gyro-Bohm scaling with \( \rho_* \) as well as a favorable positive scaling with \( \beta \). There is some error that can be introduced into the scaling calculated from regression of engineering parameters in Eq. (3), which can arise from hidden dependencies on regression variables, applicability of datapoints in various configurations (e.g. including different divertor configurations), and intrinsic errors from including data obtained from a single machine. Additional uncertainties are compounded by transforming the scaling into dimensionless parameters using Eqs. (4).

Specifically, the uncertainties in the exact fraction of RF power coupling lead to uncertainties in the estimate for loss power, and therefore error in the regression exponent \( \alpha_P \) can lead to uncertainties in all transformed dimensionless parameter exponents. Given some percent error in the calculated loss power exponent, \( \alpha_P \), Fig. 1 shows how the error propagates primarily to the \( \alpha_\beta \) exponent in the dimensionless parameter scaling, but can also contribute to a small uncertainty in the \( v_c \) and \( \rho_* \) exponents. An uncertainty of 10% in the exponent on power leads to a 35% uncertainty in the \( \beta \) exponent.
Given the issues and uncertainties with database regressions, and to more directly obtain dependencies, dimensionless parameter scans for $\rho^*$, $\nu_c$, and $\beta$ were performed in I-mode plasmas in the C-Mod tokamak. The scan holds size, shape and safety factor constant. I-mode target plasmas had safety factors of $q_{95} = 3.5 \pm 0.05$ in a lower single null shape with the $\nabla B$ drift away from the active X-point (favorable for I-mode access). Auxiliary RF power and the toroidal magnetic field were used as the primary actuators to control the temperature and plasma stored energy, $W_{MHD}$. Dimensionless quantity scaling estimates are shown in Table 1.

Table 1: Theoretical dimensionless parameter scalings with fixed geometry and safety factor. Experimental C-Mod parameter targets were designed with these B dependencies, using plasma stored energy as a proxy for temperature, which are both assumed to be dependent upon the loss power.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>I</th>
<th>n</th>
<th>$T(P_{loss})$</th>
<th>$W(P_{loss})$</th>
<th>$B$</th>
<th>$\tau$</th>
<th>$P_{loss}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho^*$</td>
<td>B</td>
<td>$B^{4/3}$</td>
<td>$B^{2/3}$</td>
<td>$B^2$</td>
<td>$\rho^*^{-2/3}$</td>
<td>$B^{-1} \rho^*_p$</td>
<td>$B^3 \rho^*_{-\alpha_p}$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>B</td>
<td>$B^4$</td>
<td>$B^2$</td>
<td>$B^6$</td>
<td>$\beta^{-4}$</td>
<td>$B^{-1} \beta^\alpha \beta$</td>
<td>$B^7 \beta_{-\alpha \beta}$</td>
</tr>
<tr>
<td>$\nu_c$</td>
<td>B</td>
<td>-</td>
<td>$B^2$</td>
<td>$B^2$</td>
<td>$\nu_c^{-4}$</td>
<td>$B^{-1} \nu_c^\alpha \nu$</td>
<td>$B^3 \nu_c_{-\alpha \nu}$</td>
</tr>
</tbody>
</table>
The electron density was varied as a function of toroidal magnetic field using gas puffing, and the plasma stored energy was used as a proxy for temperature and assumed to be a function of loss power. The magnetic field values were chosen to provide a range of \( \rho \in [0.006,0.008] \), providing sufficient sensitivity to calculate energy confinement time scaling while maintaining similar RF power coupling to the plasma. The experiments were planned based on the estimated scalings from Table 1, but the achieved engineering and plasma parameters for the dimensionless parameter scans in experiment are shown in Table 2.

Table 2: Experimental engineering and plasma parameters in I-mode dimensionless parameter scans.

<table>
<thead>
<tr>
<th>Discharge No. Time (ms)</th>
<th>( I_p ) (MA)</th>
<th>( B_T ) (T)</th>
<th>( P_{aux} ) (MW)</th>
<th>( &lt;n_{e,20}&gt; ) ( (m^{-3}) )</th>
<th>( T_{e0} ) (keV)</th>
<th>( T_{i0} ) (keV)</th>
<th>( W_{MHD} ) (kJ)</th>
<th>( \Delta t ) (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1160920007.0800</td>
<td>1.10</td>
<td>5.6</td>
<td>3.5</td>
<td>0.90</td>
<td>5.0</td>
<td>3.4</td>
<td>132</td>
<td>79</td>
</tr>
<tr>
<td>1160920007.0902</td>
<td>1.10</td>
<td>5.6</td>
<td>3.5</td>
<td>0.93</td>
<td>4.5</td>
<td>3.4</td>
<td>128</td>
<td>98</td>
</tr>
<tr>
<td>1160920010.0950</td>
<td>1.2</td>
<td>6.0</td>
<td>4.3</td>
<td>0.95</td>
<td>5.3</td>
<td>4.2</td>
<td>145</td>
<td>150</td>
</tr>
<tr>
<td>1160920012.0850</td>
<td>0.94</td>
<td>4.9</td>
<td>2.0</td>
<td>0.96</td>
<td>4.1</td>
<td>2.6</td>
<td>92</td>
<td>250</td>
</tr>
<tr>
<td>1160920017.0800</td>
<td>1.2</td>
<td>5.9</td>
<td>2.5</td>
<td>1.1</td>
<td>4.1</td>
<td>3.0</td>
<td>124</td>
<td>200</td>
</tr>
<tr>
<td>1160920017.1050</td>
<td>1.2</td>
<td>5.9</td>
<td>3.0</td>
<td>1.2</td>
<td>4.4</td>
<td>3.1</td>
<td>135</td>
<td>140</td>
</tr>
<tr>
<td>1160920026.0800</td>
<td>1.1</td>
<td>5.8</td>
<td>3.4</td>
<td>1.0</td>
<td>5.2</td>
<td>3.5</td>
<td>144</td>
<td>95</td>
</tr>
<tr>
<td>1160920026.0923</td>
<td>1.1</td>
<td>5.8</td>
<td>3.5</td>
<td>1.0</td>
<td>4.9</td>
<td>3.8</td>
<td>142</td>
<td>267</td>
</tr>
<tr>
<td>1160920027.0911</td>
<td>1.1</td>
<td>5.8</td>
<td>3.8</td>
<td>1.1</td>
<td>4.9</td>
<td>3.8</td>
<td>148</td>
<td>40</td>
</tr>
<tr>
<td>1160920028.0750</td>
<td>1.2</td>
<td>6.1</td>
<td>3.7</td>
<td>1.0</td>
<td>4.6</td>
<td>3.5</td>
<td>143</td>
<td>250</td>
</tr>
<tr>
<td>1160920028.1100</td>
<td>1.2</td>
<td>6.1</td>
<td>4.2</td>
<td>1.1</td>
<td>4.7</td>
<td>3.8</td>
<td>148</td>
<td>90</td>
</tr>
<tr>
<td>1160920029.0830</td>
<td>0.90</td>
<td>4.6</td>
<td>3.0</td>
<td>0.73</td>
<td>4.5</td>
<td>2.9</td>
<td>102</td>
<td>108</td>
</tr>
<tr>
<td>1160920029.1050</td>
<td>0.90</td>
<td>4.6</td>
<td>3.4</td>
<td>0.71</td>
<td>4.9</td>
<td>3.0</td>
<td>108</td>
<td>140</td>
</tr>
</tbody>
</table>

I-mode averaging windows on the order of 40-250ms during stationary periods \( (\dot{W} = 0) \) were chosen for calculating dimensionless quantities and energy confinement times. The scaling for each individual dimensionless parameter was performed by holding two variables constant while varying the third, as shown for two discharges (116092010/12) scanning collisionality in Fig. 2.
Power law fits for confinement time scaling with engineering parameters of auxiliary power, line average density, toroidal magnetic field, and plasma current for 13 time slices are shown in Fig. 3. It is clear there is a slight power degradation similar but weaker than H-modes and positive trend for line average density with I-mode energy confinement. The small dependence (if any) of I-mode energy confinement time with toroidal magnetic field and plasma current is also in agreement with previous single scan measurements [17], but differs from the database regression, which estimates similar exponents but with smaller uncertainties.
Fig. 3: I-mode energy confinement time scaling with a) auxiliary power, b) line average density, c) toroidal magnetic field and d) plasma current. A slight degradation of energy confinement time with $P_{aux}$ is observed, while confinement is seen to increase with $\bar{n}_e$. Little correlation with $B_T$ and $I_p$ is measured.

Confinement times were calculated using a range of estimates for the RF coupling fraction, $f_{RF}$. Though the variation of RF power coupling as a function of radius can play a role, confinement times were calculated assuming a total coupling fraction, $f_{RF}$, ranging from 80-90%. The confinement time varied 3-5% from introducing this variation in $f_{RF}$. Central ion temperature data were obtained from x-ray imaging crystal spectroscopy [19-20] and volume averaged electron temperature and density quantities from the Thomson Scattering system [21] on C-Mod to calculate the dimensionless parameters and thermal energy confinement times. Neoclassical resistivity is assumed for estimating $Z_{eff}$, and hence the impurity fraction. The experimental dimensionless parameters for the dataset are reported in Table 3.
Table 3: Experimental plasma parameters in C-Mod I-mode dimensionless scans.

<table>
<thead>
<tr>
<th>Discharge No. Time (ms)</th>
<th>$\rho_*$ (data subset)</th>
<th>$\beta$ (data subset)</th>
<th>$\nu_C$ (data subset)</th>
<th>$B\tau_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1160920007.0800</td>
<td>0.0068</td>
<td>0.82</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>1160920007.0902</td>
<td>0.0069</td>
<td>0.79</td>
<td>0.15</td>
<td>0.10</td>
</tr>
<tr>
<td>1160920010.0950</td>
<td>0.0072</td>
<td>0.80</td>
<td>0.12</td>
<td>0.10</td>
</tr>
<tr>
<td>1160920012.0850</td>
<td>0.0069</td>
<td>0.77</td>
<td>0.27</td>
<td>0.15</td>
</tr>
<tr>
<td>1160920017.0800</td>
<td>0.0061</td>
<td>0.69</td>
<td>0.20</td>
<td>0.19</td>
</tr>
<tr>
<td>1160920017.1050</td>
<td>0.0062</td>
<td>0.76</td>
<td>0.19</td>
<td>0.16</td>
</tr>
<tr>
<td>1160920026.0800</td>
<td>0.0068</td>
<td>0.83</td>
<td>0.14</td>
<td>0.13</td>
</tr>
<tr>
<td>1160920026.0923</td>
<td>0.0070</td>
<td>0.84</td>
<td>0.18</td>
<td>0.13</td>
</tr>
<tr>
<td>1160920027.0911</td>
<td>0.0070</td>
<td>0.87</td>
<td>0.13</td>
<td>0.16</td>
</tr>
<tr>
<td>1160920028.0750</td>
<td>0.0064</td>
<td>0.74</td>
<td>0.14</td>
<td>0.15</td>
</tr>
<tr>
<td>1160920028.1100</td>
<td>0.0066</td>
<td>0.79</td>
<td>0.15</td>
<td>0.12</td>
</tr>
<tr>
<td>1160920029.0830</td>
<td>0.0076</td>
<td>0.92</td>
<td>0.06</td>
<td>0.08</td>
</tr>
<tr>
<td>1160920029.1050</td>
<td>0.0079</td>
<td>1.0</td>
<td>0.06</td>
<td>0.07</td>
</tr>
</tbody>
</table>

III. Results: Estimated Energy Confinement Time Scaling with $\rho_*$ for C-Mod I-mode Plasmas

In principle, choosing a data subset with similar values of $\nu_C$ and $\beta$ should allow a scaling exponent for $\rho_*$ to be calculated from experiment assuming negligible effects from the other dimensionless parameters. The sensitivity of this assumption is examined by doing the same calculation on three different data subsets. The range of dimensionless parameter values summarized in Table 3 are visualized in Fig. 4. Three groups are defined at different values of collisionality, where several time points have well matched values of $\nu_C$ and $\beta$.  

9
Figure 4: Phase space of dimensionless quantities $\nu_C$ and $\beta$ showing the range of each variable and defining data subsets $A$, $B$, and $C$ used for calculating the energy confinement scaling with $\rho_*$. 

The thermal energy confinement times are used for all scaling calculations, though Fig. 5 shows that the thermal (taking into account integrals of plasma profiles) and MHD normalized energy confinement times exhibit similar trends. This example is for the $\rho_*$ scan, though similar results are observed for $\beta$ and $\nu_C$. 

Normalized Confinement Time Scaling with $\rho_*$

- MHD
- Thermal
Figure 5: MHD and thermal normalized energy confinement times as a function of gyroradius are shown to exhibit similar trends.

Figure 6 illustrates the power law fits for $\rho_*$ for each data subset, and Table 4 summarizes the exponents calculated from these fits. The $\rho_*$ fit for Groups $A_\rho$ and $B_\rho$ suggest a strong scaling ($\alpha_\rho = -5.0, -4.5$), indicating plasma transport greater than gyro-Bohm form ($\alpha_\rho \approx -3$). The exponent from Group $C_\rho$ indicates a more modest scaling ($\alpha_\rho = -2.2$), but it should be noted that there are only two data points in this group that have a smaller range in $\rho_*$ compared to the other two subsets. The average and standard deviation of the exponential fits from the three data subsets are used to estimate the energy confinement time scaling $B\tau_E \propto \rho_*^{-3.9\pm1.5}$. These results, in conjunction with the reformed $\rho_*$ scaling from the database study in reference 11 that was derived in Section II ($\alpha_\rho = -3.7$), show accumulating evidence that the I-mode energy confinement time scaling is gyro-Bohm in nature similar to ELMy H-modes, or most conservatively interpreted as greater than Bohm transport.

Figure 6: Three exponential fits of normalized thermal energy confinement time as a function of ion gyroradius. Group $A_\rho$ is represented by the red circles (solid line), Group $B_\rho$ by green squares (dashed line), and Group $C_\rho$ by the blue triangles (dotted line).
IV. Results: Estimated Energy Confinement Time Scaling with $\nu_C$ and $\beta$ for C-Mod I-mode Plasmas

Similar methods can be used to calculate the energy confinement time scaling with $\nu_C$ and $\beta$, though these variables are non-dominant in the sensitivity of the confinement time in this dataset. Table 5 shows how tightly correlated the dimensionless variables are, with the most significant correlation between $\rho_*$ and $\beta$. The strong exponent calculated for $\rho_*$ indicates that even slight variation will lead to significant uncertainty in the $\nu_C$ and $\beta$ exponents. With this dataset, it may be interpreted that only the estimate for the $\rho_*$ exponent is meaningful, but the results for these two other dimensionless variables are still presented in this section for completeness.

Table 5: Correlation matrix between dimensionless quantities $\rho_*$, $\nu_C$, and $\beta$. The most significant correlation is calculated to be as large as 90% between $\rho_*$ and $\beta$.

<table>
<thead>
<tr>
<th></th>
<th>$\rho_*$</th>
<th>$\beta$</th>
<th>$\nu_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_*$</td>
<td>1</td>
<td>0.9</td>
<td>-0.65</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9</td>
<td>1</td>
<td>-0.73</td>
</tr>
<tr>
<td>$\nu_C$</td>
<td>-0.65</td>
<td>-0.73</td>
<td>1</td>
</tr>
</tbody>
</table>

The range of the dimensionless parameter space obtained in experiment is shown in Fig. 7. Similar to the $\rho_*$ calculation, several data subsets are defined where the other dimensionless parameters are reasonably matched. The coefficients from the exponential fits for the data groups defined in Fig. 7 are reported in Table 6. The exponent calculated for $\nu_C$ is shown to be small and positive for all groups. The significant variation in the $\beta$ exponent leads to an experimental uncertainty that cannot resolve the sign of the exponent as positive or negative, which yields an inconclusive result for energy confinement time scaling with $\beta$. 

<table>
<thead>
<tr>
<th>Scan Variable</th>
<th>Group A</th>
<th>Group B</th>
<th>Group C</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_*$</td>
<td>-5.0</td>
<td>-4.5</td>
<td>-2.2</td>
<td>$-3.9 \pm 1.5$</td>
</tr>
</tbody>
</table>
Figure 7: Phase space of dimensionless quantities defining data subsets for calculating the energy confinement time scaling with $\nu_C$ and $\beta$.

Table 6: Calculated power law exponents for each data grouping for $\nu_C$ and $\beta$ assuming negligible effects from other dimensionless parameters.

<table>
<thead>
<tr>
<th>Scan Variable</th>
<th>Group A</th>
<th>Group B</th>
<th>Group C</th>
<th>Group D</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>2.2</td>
<td>5.5</td>
<td>-1.3</td>
<td>-0.76</td>
<td>1.4 ± 3.1</td>
</tr>
<tr>
<td>$\nu_C$</td>
<td>0.55</td>
<td>0.17</td>
<td>0.60</td>
<td>x</td>
<td>0.44 ± 0.24</td>
</tr>
</tbody>
</table>

V. Discussion

A. Multiple Regression Analysis

To assess the significance of each dimensionless variable, multiple linear regression analysis is employed to calculate a best fit to the experimental energy confinement times considering the three dimensionless variables $\rho_*$, $\beta$, and $\nu_C$ using the 13 point dataset.

$$\Omega \tau_E \propto \rho_*^{3.34\pm1.2} \beta^{1.34\pm1.1} \nu_C^{0.34\pm0.15}$$ (6)

This approach has some caveats due to the correlation between variables as shown in the previous section, so the dataset was decomposed into three principal components, then regressed using this mostly orthogonal system before transforming back into the dimensionless variables $\rho_*$, $\nu_C$, and $\beta$. This technique has been used for both H-mode and L-mode confinement time regressions in previous studies [22-23]. The principal components (PCs) are vectors derived from singular value decomposition of the dimensionless variables originally assumed to
represent a more orthogonal basis. The linear decomposition of the PCs with respect to $\rho_*$, $\beta$, and $v_C$ are shown in Figure 8. Additionally, the first principal component is comprised of approximately 98% of the original predictor variable $\rho_*$, and accounts for roughly 97% of the variance of the predictor variables in the dataset. This indicates that the $\rho_*$ variable dominates the sensitivity in the dimensionless parameter scan.

![Principal Component Variance Contribution](image)

**Figure 8:** Variance explained by each principal component in partial least squares regression of normalized energy confinement time. About 97% of the variance can be explained by the first principal component, which is dominantly comprised of $\rho_*$, indicating $\rho_*$ is the most sensitive parameter in this dataset by a significant margin.

A probability of significance for each parameter can be calculated by performing several regression analyses and assembling an f distribution probability density function to determine the likelihood of a given outcome [24]. The probabilities of significance are calculated using multiple linear regression with the primary independent variables [$\rho_*, \beta, v_C$] (not principal components) on the dependent variable $\Omega \tau_E$ to be [0.97, 0.74, 0.95]. The energy confinement time is seen to be most sensitive to the gyroradius independent variable, followed in significance by collisionality and exhibiting questionable significance to $\beta$. Due to the strong correlation between variables, it is likely that the high probability of significance for $\beta$ and possibly $v_C$ are due to their dependence on $\rho_*$. The regression fit yields an F test value of 10.4, which passes the F test to reject the null hypothesis for both the 95% ($10.4 \notin [0, 4.3]$) and 99% ($10.4 \notin [0, 8.0]$) confidence.
intervals. These results agree with the assessment that \( \rho_* \) dominates the sensitivity of the dataset from the principal component analysis.

Assessing the significance values of the three dimensionless parameters shows that the energy confinement time is preferentially sensitive to \( \rho_* \) and somewhat sensitive to \( \nu_C \), but that the exponent on \( \beta \) as an independent variable is likely to have large errors, which is reflected by the calculated uncertainty being nearly equivalent in value to the exponent itself. This multiple regression fit has a large uncertainty calculated for the \( \beta \) exponent, though it does imply a positive exponent which is more information than could be obtained from the individual parameter fit in Section IV. To investigate the \( \beta \) scaling in further, more experiments or a different dataset are required.

Reversing the transformation in Eqs. (4), the dimensionless expression derived from multiple regression analysis can be re-dimensionalized. The transformation is calculated for the range of exponents (100 within the error bar) for each variable from Eq (6). The exponents in Table 7 are the mean of all the values calculated from the transformation and the uncertainty is one standard deviation. One calculation was performed assuming \( \alpha_q = 0 \), and the second assuming \( \alpha_q = -0.72 \) taken from the database regression from Eq. (5). The calculated values of dimensional exponents are consistent with the dedicated experimental scans in that the density scaling is positive, though somewhat stronger in this calculation, there is a modest power degradation, and small or inconclusive exponent in plasma current and magnetic field as interpreted by the large error bars.

Table 7: Back calculation of engineering parameter scaling exponents from regression on dimensionless variables. Exponents are consistent with confinement time scalings from dedicated scans in reference 16 except for a slightly larger dependence on density.

<table>
<thead>
<tr>
<th>Case</th>
<th>( \alpha_n )</th>
<th>( \alpha_p )</th>
<th>( \alpha_I )</th>
<th>( \alpha_B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_q = 0 )</td>
<td>1.57 ± 0.82</td>
<td>−0.39 ± 0.42</td>
<td>0 ± 0</td>
<td>−0.64 ± 1.7</td>
</tr>
<tr>
<td>( \alpha_q = -0.72 )</td>
<td>1.57 ± 0.82</td>
<td>−0.39 ± 0.42</td>
<td>0.44 ± 0.33</td>
<td>−1.10 ± 2.19</td>
</tr>
</tbody>
</table>

B. Comparison of Scaling Estimates

The I-mode energy confinement time scaling yields varied results using the different methods. Within the groups defined to fit \( \rho_* \) exponents as a single parameter, experimental
values of $\beta$ and $\nu_C$ varied on the order of 3-8% of the mean value. For the single parameter scans for $\beta$ and $\nu_C, \rho_*$ varied on the lower end of this scale with about 4-5% deviation from the mean. The similar exponents calculated for $\rho_*$ and $\nu_C$ between the one dimensional technique and the MRA analysis yield further confidence in the fits for these variables. The MRA power scaling law in Fig 9 is shown to fit the experimental confinement data with $R^2 = 0.98$. The additional confinement data from reference 11 could not be added to the fit in Fig. 9 because of the lack of ion data required to calculate $\rho_*$ quantities.

![Normalized Confinement Time Experimental Scaling](image)

**Fig 9**: Experimental energy confinement time versus power law scaling calculated from a multiple regression fit for dimensionless parameters $\rho_*, \beta$, and $\nu_C$

Table 8 compares the exponents for the energy confinement time in I-mode plasmas to those previously calculated for L-mode and H-mode plasmas. The ITER scaling laws [6] use $\nu_\ast \propto nRT^{-2} q e^{-3/2}$ as a dimensionless quantity instead of $\nu_C$, with $Z_{eff} = 1$ assumed for the entire database. Therefore Table 8 is not a direct comparison of the collisionality exponent, though since the I-mode dataset in this investigation has no variation in $q$, the trends in $\nu_C$ should correlate almost exactly with $\nu_\ast$.

Table 8: Power law coefficients for energy confinement time scalings for various plasma regimes. The I-mode database fit is a reformed dimensionless parameter scaling law derived from a dimensional scaling calculated from a C-Mod I-mode database with various shapes, divertor configurations, and plasma parameters. The experiment 1D fits are derived from
dedicated scans of dimensionless parameters described in Sections III and IV. The experiment MRA fits include all the data points from the dataset described in Section V.

<table>
<thead>
<tr>
<th>Confinement Fit</th>
<th>( \rho^* )</th>
<th>( \beta )</th>
<th>( \nu )</th>
<th>( q_{95} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ITER H98</td>
<td>-2.7</td>
<td>-0.9</td>
<td>0.08</td>
<td>-3</td>
</tr>
<tr>
<td>ITER L97</td>
<td>-1.85</td>
<td>-1.41</td>
<td>0.21</td>
<td>-3.6</td>
</tr>
<tr>
<td>I-mode derived database</td>
<td>-3.7 ± 0.2</td>
<td>0.59 ± 0.24</td>
<td>-0.61 ± 0.03</td>
<td>-0.72 ± 0.15</td>
</tr>
<tr>
<td>Experiment 1D</td>
<td>-3.9 ± 1.5</td>
<td>1.4 ± 3.1</td>
<td>0.44 ± 0.24</td>
<td>x</td>
</tr>
<tr>
<td>Experiment MRA</td>
<td>-3.34 ± 1.2</td>
<td>1.34 ± 1.1</td>
<td>0.34 ± 0.15</td>
<td>x</td>
</tr>
</tbody>
</table>

The \( \rho^* \) scaling consistently suggests gyro-Bohm type transport \( (\alpha_{\rho} \approx -3) \) from multiple sets of data and means of calculation. This, in conjunction with the high level of sensitivity of the presented dataset to the \( \rho^* \) parameter, suggests that the I-mode energy confinement time scales similarly to H-mode plasmas.

There is a large variance in exponents calculated for the \( \beta \) scaling between datasets and analysis methods, which has also been the case in the past with \( \beta \) scalings for other plasma regimes investigated on DIII-D and JET [25-28]. The single parameter scaling for \( \beta \) yields an inconclusive sign on the exponent. The MRA method suggests a positive scaling with large uncertainty and a low probability of significance of the parameter to this dataset. The I-mode database study also suggests a positive exponent, which contrasts from both L-mode and H-mode degradation with \( \beta \). These observations point towards a possible positive scaling with \( \beta \), though rather inconclusive given the uncertainties. Most I-modes operate far from MHD limits [29-30], perhaps leading to the \( \beta \) scaling being of less importance than a parameter like \( \rho^* \). The smaller \( \beta \) degradation in I-modes (correlating with a smaller power degradation [14]) is possibly due to the lack of MHD activity in both the core and edge as compared to ELMs or core tearing modes often present in conventional H-modes.

The exponents calculated for \( \nu_C \) are consistent in this dataset as small, but positive, which point towards similarities to the scaling for L-mode plasmas rather than a negligible dependence as calculated for H-modes. The database regression exponent for \( \nu_C \) is the opposite sign from the dedicated experiments, suggesting that the variations from divertor configuration or plasma shape may play a more nuanced role in confinement as a function of collisionality, and should be further investigated.
The MRA power law can be used to extrapolate the normalized energy confinement time to future devices, as shown in Fig. 10. While this is a far projection and data from more machines would be required for confirmation, it is useful to see that energy confinement time scalings in I-mode plasmas are dominated by the $\rho_*$ exponent, and therefore extrapolate similarly to ELMy H-mode plasmas. For reference, the normalized energy confinement time for ELMy H-modes in ITER is $B\tau_{et} \sim 5.3 [T] \times 5.0 [s] = 26.5$, which lies somewhere between the projections for SPARC [31] and ARC [32] in Fig. 10, using the assumed dimensionless parameters in Table 9. This is a favorable global energy confinement projection for compact, high field devices with low $\rho_*$ such as SPARC and ARC. ITER parameters accomplish similar levels of energy confinement to the ARC projection for this I-mode scaling law with lower magnetic field, but larger device size. Confinement scaling projections must also be coupled with regime access constraints. While it is not probable that I-mode will be an operating regime accessible in ITER with the planned $\nabla B$ drift directed towards the X-point, it is possible that future low $\rho_*$ devices like SPARC and ARC may be configured to access a high performance I-mode [17]. Additionally, at the power levels assumed to meet the assumed dimensionless parameter requirements for these projections, it is possible that the plasma would transition to H-mode before reaching these levels of confinement. Detailed physics based understanding of the I-
H transition is required to accurately model this energy confinement time projection to a future device.

Table 9: Assumed dimensionless parameters and extrapolated confinement time from multiple regression.

<table>
<thead>
<tr>
<th></th>
<th>( \rho_* )</th>
<th>( \beta )</th>
<th>( \nu_c )</th>
<th>( B\tau_E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPARC</td>
<td>0.002</td>
<td>1.0</td>
<td>0.015</td>
<td>5</td>
</tr>
<tr>
<td>ARC</td>
<td>0.001</td>
<td>1.9</td>
<td>0.01</td>
<td>100</td>
</tr>
<tr>
<td>ITER</td>
<td>0.001</td>
<td>2.5</td>
<td>0.01</td>
<td>144</td>
</tr>
</tbody>
</table>

C. Interpretation of Dimensionless Parameter Scalings

A strength of global energy confinement time scaling calculations is simplicity, such that detailed information about physical processes such as local temperature and density turbulence drives or atomic physics is encompassed within the data. However, the simplicity of the method introduces caveats to interpreting such an exercise. It is shown in this investigation that the \( \rho_* \) parameter is the most sensitive in this dataset, making small changes in this variable influential in interpreting the dependences of the other variables. Additionally, data used from a single machine cannot fully capture all the physics required for scaling laws to be properly projected to future devices, necessitating the inclusion of more data from various other devices to enhance confidence in the extrapolations for SPARC, ARC, and ITER. This scaling implicitly assumes that negligible energy losses occur from processes like radiation and charge exchange.

Additionally, the use of Buckingham Pi theorem requires no maximum set of variables to describe a physical system. While the dimensionless parameters chosen here have been shown to describe tokamak plasmas in the past [5], it is possible that there are additional physics considerations in I-mode plasmas such that there are hidden variables in the scaling law, or that the traditional dimensionless variables are not sensitive enough to matter in I-modes (e.g. \( \beta \)). Suggested hidden variables from previous studies [33] are rotation (Mach number) and the associated ExB shear that affects turbulence suppression, along with the radial scale lengths of density and safety factor profiles which may play a role in L-mode confinement. Since I-modes share some similar qualities to H-modes with respect to thermal transport and L-modes with respect to particle transport, both of these variables remain a topic for future investigation. I-mode access is likely governed by pedestal instability drives, and therefore more detailed studies of how edge physics quantities scale to reactor relevant plasma regimes will need to be
completed for more accurate projections (e.g. ExB shear scaling with $\rho_*$ and its impact on the weakly coherent mode drive and particle transport).

Nevertheless, the caveats of the study should not deter interpretation of the data - specifically the implications of the energy confinement time likely scaling as gyro-Bohm. Gyro-Bohm physics indicates that the local instabilities causing heat and particle transport are microturbulent in nature, and require gyro-kinetic modeling instead of standard random walk diffusion processes which would govern Bohm-like transport. Core transport simulations using GYRO [34] have indicated I-modes have ITG dominant modes, with reduced growth rates but similar ExB shearing rates to L-mode plasmas. Upon the transition to I-mode, the corresponding increase of core temperature concurrent with the rise in edge temperature indicates profile stiffness, highlighting the importance of future pedestal scaling studies. The observed decrease in core turbulence indicates an enhanced ITG drive for heat flux, consistent with gyro-Bohm dependences, which leads to the reduced $a/L_T$ observed in experiment [34]. Recent simulations [35] have shown that small scale electron temperature gradient driven instabilities dominate the heat transport in edge pedestal of C-Mod I-mode plasmas, which would be consistent with the gyro-Bohm scaling found in this study as well. Those simulations also found that the impurity particle transport in the edge is dominated by large scale ion temperature gradient driven instabilities [35-36], which lends towards L-mode like extrapolations for pedestal physics. Both of these factors can be interpreted as favorable for extrapolation to large devices in that a high field, compact tokamak like SPARC or ARC would have H-mode like energy confinement with enough particle transport to flush impurities and relax pedestal gradients below the peeling-balloonning ELM threshold. These results reinforce that I-mode is a high confinement, intrinsically ELM stable regime suitable for use as a target plasma for future fusion energy devices.

VI. Conclusions

An updated estimate of the energy confinement time scaling for the I-mode confinement regime is calculated from a dimensionless parameter scan performed on the C-Mod tokamak. The I-mode regime is a possible target scenario for future reactors due to the high temperature pedestal and intrinsic ELM stability. Therefore, it is desirable to understand how energy
transport and confinement scale with dimensionless parameters to determine if the regime is compatible with reactor scale machines.

Previous studies estimated an I-mode confinement time power law scaling with engineering parameters [11] \( I_p, B_T, \tilde{n}_e, P_{loss}, R, \kappa, \epsilon \). This scaling is based on a large dataset of C-Mod I-mode plasmas with different shapes, input powers, \( \nabla B \) drift direction, and divertor configurations. However, the dataset contained potential covariances between these parameters. The scaling can be recast into a power law scaling estimate for I-modes using dimensionless parameters \( \rho_*, \beta, \nu_C \), and \( q \). To more directly obtain these dependences on physical variables and minimize covariances, a set of dedicated experiments was designed to scan the dimensionless variables directly in I-mode plasmas holding the safety factor constant. Power law scaling coefficients are calculated of the \( \rho_* \), \( \beta \), and \( \nu_C \) dependencies using two methods: 1) single parameter scaling calculation with the remaining two variables being well-matched, 2) multiple regression analysis.

The \( \rho_* \) scaling estimate for both methods of calculating exponents suggests that the I-mode regime may have gyro-Bohm type scaling, similar to ELMy H-mode plasmas and the ITER H\textsubscript{98} scaling law, though the exponents have large enough uncertainties to within Bohm scaling (\( \alpha_{\rho_*} = -3.9 \pm 1.5 \)). The gyroradius is the dominant parameter in this dataset and the scaling extrapolates favorably to compact, high field future devices such as SPARC and ARC.

The energy confinement time scalings were also calculated for \( \nu_C \) and \( \beta \), though dataset is less sensitive to these variables. The \( \nu_C \) scaling estimate suggests that the exponent is small, but positive (\( \alpha_{\nu} = 0.44 \pm 0.24 \)), similar to the L-mode scaling for ITER, though within error bars falls close to the range of the H-mode ITER scaling for collisionality. The \( \beta \) scaling estimate varies largely in this dataset, and statistical methods show the relatively low sensitivity of the energy confinement time to this dimensionless parameter, resulting in an inconclusive estimate for the \( \beta \) scaling (\( \alpha_{\beta} = 1.4 \pm 3.1 \)). This may be a consequence of I-mode operating far from MHD stability limits, so that the \( \beta \) scaling may be assumed to be of secondary importance to the other dimensionless parameters.
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References


