High Power Long Pulse Microwave Generation from a Metamaterial Structure with Reverse Symmetry

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ABSTRACT
Experimental operation of a high power microwave source with a metamaterial (MTM) structure is reported at power levels to 2.9 MW at 2.4 GHz in full one microsecond pulses. The MTM structure is formed by a waveguide that is below cutoff for TM modes. The waveguide is loaded by two axial copper plates machined with complementary split ring resonators, allowing two backward wave modes to propagate in S-Band. A pulsed electron beam of up to 490 kV, 84 A travels down the center of the waveguide, midway between the plates. The electron beam is generated by a Pierce gun and is focused by a lens into a solenoidal magnetic field. The MTM plates are mechanically identical but are placed in the waveguide with reverse symmetry. Theory indicates that both Cherenkov and Cherenkov-cyclotron beam-wave interactions can occur. High power microwave generation was studied by varying the operating parameters over a wide range, including the electron beam voltage, the lens magnetic field and the solenoidal field. Frequency tuning with magnetic field and beam voltage was studied to discriminate between operation in the Cherenkov mode vs. the Cherenkov-cyclotron mode. Both modes were observed, but pulses above 1 MW of output power were only seen in the Cherenkov-cyclotron mode. A pair of steering coils were installed prior to the interaction space to initiate the cyclotron motion of the electron beam and thus encourage the Cherenkov-cyclotron high power mode. This successfully increased the output power from 2.5 MW to 2.9 MW (450 kV, 74A, 9% efficiency).

I. INTRODUCTION
A metamaterial (MTM) is an artificial material with novel electromagnetic properties. MTMs are often implemented as periodic structures with the period much smaller than the wavelength. Among them, MTMs with a negative group velocity, or the so-called left-handed MTMs, are especially interesting in terms of interaction with an electron beam in an active electron device due to the exotic phenomenon of reversed Cherenkov radiation, or backward Cherenkov radiation. In normal materials with a positive group velocity, the radiated waves travel forward together with the incoming beam. In a MTM with a negative group velocity, since the group velocity is anti-parallel to the phase velocity, the radiated waves travel backwards\(^1\). Reversed Cherenkov radiation has been extensively studied both theoretically and experimentally\(^2\). Ref. \(^4\) presents the mathematical solution for the Cherenkov radiation in a MTM with simultaneously negative permittivity \(\varepsilon\) and permeability \(\mu\). Ref. \(^5\) shows theoretically that two electron beams can develop an instability from the backward Cherenkov radiation when passing through a slab of left-handed MTM. Ref. \(^6\) calculates the radiation from a charge moving across the boundary between a normal medium and a left-handed MTM. Ref. \(^7\) discovers theoretically that the radiation of a charge moving in an infinite MTM structure of parallel wires concentrates near certain rays behind the charge. Ref. \(^8\) presents a way to mimic a moving charge with a phased antenna array, and the researchers measured the backward radiating pattern as an analogy to the reversed Cherenkov radiation from a real charge. Ref. \(^9\) describes an experimental measurement of the radiated signal from a 20 ns long single sheet beam electron bunch traveling through a MTM. They observed that the radiated power in the backward direction was stronger than that in the forward direction.

Several active MTM-based structures with an electron beam for high power microwave generation have been studied theoretically\(^{10-18}\). However, very few experiments have been carried out to actually generate high power microwaves with an electron beam passing through an MTM structure\(^{19-23}\). At MIT, in our previous experiment\(^{19}\), we built a structure with two
MTM plates loaded in a waveguide with dimensions below the cut-off of the TM_{11} mode. The implementation we chose is the complementary split ring resonators (CSRRs) [24]. The CSRRs enable us to design an all-metal structure compatible with the vacuum condition and the high power requirements. The CSRRs are responsible for a negative permittivity \( \varepsilon \), and the below-cut-off waveguide for the TM mode offers a negative permeability \( \mu \) [25], so microwaves with a negative group velocity can propagate in the structure. In our previous design, the two MTM plates are identical and they are arranged in a symmetric way. There are two types of eigenmodes in the structure, a symmetric mode with a longitudinal electric field on axis, and an antisymmetric mode with a transverse electric field on axis. In the experiment, megawatt power level pulses were generated in the antisymmetric mode in the Cherenkov-cyclotron type of interaction at a low magnetic field. The Cherenkov-cyclotron mode is also called the anomalous Doppler mode [26-28]. Surprisingly, high power was only found in the Cherenkov-cyclotron mode and was not found in the expected Cherenkov mode. Although multi-megawatt power level pulses were obtained with the symmetric structure, the microwave pulses had a short pulse length. With a continuous electron beam 1 \( \mu \)s long, microwave pulses of only 100 ns to 400 ns long were observed. In this paper, we will present experimental results of full 1 \( \mu \)s long, multi-megawatt microwave pulses achieved by changing the symmetric structure into one with reverse symmetry.

II. CIRCUIT DESIGN: REVERSE MTM (MTM-R)

The idea of a MTM structure with reverse symmetry comes from our experience with the previous experiment with the symmetric structure. In that structure, the electron beam favors the antisymmetric mode with a transverse electric field on the beam axis, and the symmetric mode with a longitudinal electric field on axis does not help to generate high power. So in the new design, we have reversed one of the MTM plates to create asymmetry. This new structure is named the reverse MTM structure, or the MTM-R structure. In this way, the symmetric mode with a longitudinal electric field on the beam axis, which does not generate high power, is eliminated.

The experiment was performed at MIT with the test facility shown in Fig. 1. The electron beam is emitted from a Pierce-type electron gun. The hot gun cathode is pulsed by 1-\( \mu \)s high voltage pulses up to 490 kV, and the emitted beam current at 490 kV is 84 A. The electron beam is then confined by two magnets. The one closer to the gun is referred to as the magnetic lens, and the one farther away from the gun and overlapping the MTM circuit region is referred to as the solenoid magnet. The field of the two magnets can be adjusted independently and continuously from zero to their maximum values of 840 G for the lens and 1500 G for the solenoid. After the beam passes through the interaction region, it is dumped into the collector, and the microwaves generated in the MTM structure are guided through two waveguides, which we call Arm 1 and Arm 2, with calibrated Bethe hole couplers to RF loads. The couplers pick up a small amount of power in the WR284 waveguide with a coupling coefficient of -64 dB for Arm 1 and -61 dB for Arm 2.

The MTM-R structure is shown in Fig. 2. It is a stainless steel waveguide with two copper MTM plates brazed onto it. The C-shaped cuts on the two MTM plates are aligned in opposite directions; that is, one plate is “reversed” from the other, as can be seen in Fig. 2. The period of the MTM plate, containing one C-shaped cut, is 10 mm. The length of one period is much smaller.

Fig. 1 Experimental setup.
Fig. 2 MTM-R circuit design. Two copper plates with the C-shapes aligned in opposite directions are placed in a rectangular waveguide. The period of the MTM plates is 10 mm. The red cylinder represents the electron beam traveling in the $+z$ direction along the central axis of the waveguide. The origin of the coordinate system is placed in the center of the waveguide, and $z = 0$ is at the edge of the MTM plates.

Fig. 3 Dispersion curves from CST eigenmode simulation for a period of the MTM structure. The Cherenkov and the Cherenkov-cyclotron beamlines are calculated for a 490 kV electron beam in a constant magnetic field of 400 G.

The electric field distribution for the two modes is shown in Fig. 4 (a), and the field is plotted on the middle cutting plane $x = 0$ in one period containing one pair of C-shaped cuts. Both Mode 1 and Mode 2 are hybrid. Fig. 4 (b) shows the phase of the transverse electric field $E_y$, and Mode 1 and Mode 2 have different relative phases. The waves generated in the upper and lower parts of the waveguides are guided into two S-band waveguide bends, Arm 1 and Arm 2, with Arm 1 on the bottom and Arm 2 on the top according to Fig. 2.

An example of the simulated power in the two output couplers is presented in Fig. 5 (b). The power levels in the two output ports are uneven in the MTM-R design. This is different from the symmetric MTM structure in Ref. [19], with the symmetry between the two arms broken in this experiment. The phenomenon of uneven power levels is caused by the fact that the longitudinal locations of peak electric field are alternating between the two plates as the beam spirals. So with a finite length of the MTM waveguide, one arm can receive the majority of the generated microwave power. A plot of the transverse electric field on the middle cutting plane $x = 0$ is shown in Fig. 5 (c), and it explains why Arm 1 receives more power in the example illustrated. The power splitting between the two arms varies with the structure length, and this has been verified with a series of PIC simulations on structures with different numbers of longitudinal periods.
Fig. 4 Electric field distribution of Mode 1 and Mode 2. (a) E field on the middle cutting plane of $x = 0$ for one period of the structure. (b) Phase of the $E_y$ field for Mode 1 and Mode 2. The shaded blocks denote the positions of the two MTM plates.

III. EXPERIMENT

3.1 Methods of measurement

The power and frequency of the microwave pulses were measured respectively with a power meter and a fast oscilloscope. Care was taken to assure that the power measurement with the Bethe hole coupler only measured the power in the operating mode at 2.4 GHz. Small amounts of power at higher frequency mainly at higher harmonics also can couple into the hole coupler but was carefully filtered out using low pass and bandpass filters. A mixer
was also used to extend the measurable higher harmonics frequency to 15 GHz using a signal generator as a local oscillator. With the mixer measurement, we observed up to the 6th order harmonics of the fundamental mode, while later, in measuring the power at 2.4 GHz, all these higher harmonics were filtered out correctly with the low pass filters.

3.2 High power operation

The power and frequency measurements of the device were done in a large 3D parameter space, with the three parameters being the beam voltage, the lens field and the solenoid field. For the Pierce type gun, the beam current scales with the beam voltage, and thus it is not an independent parameter.

In the 3D parameter space of voltage-lens-solenoid, we observed in some regions full-length, one microsecond microwave pulses with a few megawatts of power. The high power region in the lens-solenoid 2D space with a fixed beam voltage is shown in Fig. 6 (a). In the half-bounded region between the two boundary lines, marked “high power” in the figure, the output power is higher than 1 MW. However, the power drops abruptly to the level of 1 kW outside of the high power region. The boundary contour denotes the critical condition for the megawatt level power to start. The contour shrinks with a lower beam voltage. In the high power region, the operating voltage (490 kV, 460 kV, or 420 kV) is higher than the critical voltage; while outside of the bounded region, the operating voltage is below the critical voltage. So the shrinking high power region indicates that the critical voltage varies with both the lens and the solenoid field. Fig. 6 (b) then shows measurement of the critical voltage at some lens field values. The trend is in agreement with the information in Fig. 6 (a) that with a lower operating voltage, the high power operation space shrinks.

The occurrence of regions of megawatt level power as shown in Fig. 6 has several reasons. The first reason is that the generated high power is from the Cherenkov-cyclotron instability. This instability arises only when the solenoid field is below a certain threshold so that the cyclotron motion wins over the longitudinal bunching. The second reason is that the beam quality is varied at different conditions. Normally we would assume the magnetic field only matters in the interaction region, but from Fig. 6 we notice the effect of the lens magnet. The lens field, together with the solenoid field, decides the beam radius and scalloping through the space charge effect. Two examples of the beam profiles are presented in Fig. 7. In the experiment, high power was measured with the condition of Fig. 7 (b), but not Fig. 7 (a). With the same lens field, a higher solenoid field in Fig. 7 (a) leads to a mismatched beam with a smaller initial radius at the structure entrance and heavier scalloping compared to the low solenoid field in Fig. 7 (b). The beam radius and scalloping are the two aspects that make the difference between high power and low power in the two cases.

On the one side, the beam radius should be reasonably large to fill the space between the two MTM plates. The MTM plates support surface waves concentrated on the plates, so the electron beam under a lower magnetic field sees a stronger electric field near the plates resulting in a stronger interaction. But when the magnetic field is too low, beam interception happens before the microwave mode is excited so the effective beam current is reduced and high power cannot be generated.
On the other side, scalloping is not favorable to high power generation, and this has been illustrated in the CST PIC simulations. If the scalloping before the beam enters the MTM waveguide is neglected in the PIC simulation, the code predicts high power from longitudinal bunching with a high magnetic field of over 1 kG. However, when scalloping is taken into account, the saturation time of high power lengthens from below 300 ns to above 600 ns. With a finite high voltage pulse length on the electron gun, a longer saturation time makes it more difficult to excite high microwave power.

3.3 Output frequency

Among the megawatt and microsecond long pulses, three categories of pulses were observed. In the first category, the two output arms have the same microwave frequency; while in the second category, the measured frequencies in the two arms differ by about 30 MHz. In these two categories, each arm has a single frequency indicating coherent radiation from the electron beam. In the third category, multiple frequencies are measured in one or both arms.

Fig. 8 is an example of the first category where both arms have the same frequency. Fig. 8 (a) shows the gun voltage trace and the collector current trace. The beam voltage is 490 kV, and the collector current goes up and then drops to zero quickly. The full current at the gun is 84 A, and the missing collector current is from beam interception on the structure. This is as expected since the design modes are hybrid with a transverse deflecting electric field. However, the MTM-R structure is comprised of copper plates of 3 mm thickness and there was no evidence of arcing or damage to the plates. Fig. 8 (b) shows the power traces. The flat-top power is 1.5 MW, so the efficiency is 4%. The power rises from almost zero to the full value within a very short time, just 100 ns. The operating lens field is 725 G and the solenoid field is 339 G. Fig. 8 (c) shows the measured voltage spectra for the microwaves in Arm 1 and Arm 2. Both arms see the same single frequency of 2.37 GHz. The radiated waves are coherent with a bandwidth of only a few MHz. The signal in Arm 2 has a small side band at 2.39 GHz, but it is 12 dB down in power from the main peak at 2.37 GHz since the spectrum represents the relative voltage.

Fig. 9 is an example of the second category where the two arms each have a single but different frequency. The peak voltage is 420 kV, and the full beam current at the gun is 65 A. The total output power is 2.5 MW with a full 1 µs pulse width; the efficiency is 9%. The lens field is 725 G and the solenoid field is 437 G. The second category has the flattest output power traces and the highest energy efficiency. Observation of this category is always under the condition that the operating voltage is above but very close to (within 20 kV) the critical voltage. The reason is that there are several modes with different interaction types possibly happening in the structure, as stated in Section II. These modes all have various critical voltages, so when the operating voltage is just above the lowest threshold, only one type of interaction leads to high power microwaves, and the pulse shape is flat. As the voltage goes higher, mode competition occurs, so the third category of pulses are observed, with multiple frequencies and a messy pulse shape.

In the third category, multiple frequencies are generated in the microwave pulses. A sample pulse in this category with a high peak microwave power is shown in Fig. 10. The beam voltage is 475 kV, and the peak collector current before beam interception happens is 60 A. The peak power is 8 MW though it happens in a short burst. Mode competition exists in this category, when the operating voltage is far away from the critical voltage of high power in any mode.
Fig. 8 Sample pulse in the first category, with high power microwaves in both arms at the same coherent frequency of 2.37 GHz. The operating lens field is 725 G and the solenoid field is 339 G. (a) Voltage and collector current traces. (b) High power microwave traces. (c) Normalized voltage spectra of the microwaves in the two arms.

Fig. 9 Sample pulse in the second category, with high power microwaves in both arms each at a single but different frequency. The operating lens field is 725 G and the solenoid field is 437 G. (a) Voltage and collector current traces. (b) High power microwave traces. (c) Normalized voltage spectra of the microwaves in the two arms.
3.4 Frequency tuning

By analyzing the frequency tuning with the beam voltage and the magnetic field, we can determine the type of beam–wave interaction, since the frequency dependence varies for different modes and interaction types.

Fig. 11 shows the frequency dependence on the beam voltage under different magnetic field combinations. For both Fig. 11 (a) and Fig. 11 (b), the lens value is 729 G, but the solenoid field is 648 G and 1511 G, respectively. We can see that the frequency tuning with voltage fits the Cherenkov-cyclotron type and the Cherenkov type of interaction, respectively. The fitted Cherenkov-cyclotron theory line in Fig. 11 (a) has a larger slope than the
Cherenkov theory line in Fig. 11 (b), since in the Cherenkov-cyclotron dispersion $\omega = k_z v_z - \Omega_c / \gamma$, both terms on the right-hand side increase with a higher beam energy.

Fig. 12 shows the frequency dependence vs. solenoid field. The Cherenkov frequency is not affected by the magnetic field, while the Cherenkov-cyclotron frequency decreases with a higher field. Both types are observed in the experiment. The high power is always in the Cherenkov-cyclotron type of interaction below 475 G. And in the low power region, the Cherenkov-cyclotron mode is observed below 830 G and the Cherenkov interaction is observed above 830 G. This feature also agrees with the mode selection based on the measurement in Fig. 11.

### 3.5 Improved operation with steering coils

The idea of applying steering coils comes from the fact that high power is always observed in the deflecting field mode with the cyclotron motion involved. We added a transverse steering magnetic field to give the beam a modest transverse kick with the hope that it would initiate the higher-power Cherenkov-cyclotron instability. Also, pushing the beam closer to one of the plates can increase the field amplitude witnessed by the beam, thus improving the beam-wave interaction.

The steering coils, built by Haimson Research Corp., are shown in Fig. 13. The coils consist of two pieces facing together providing a transverse magnetic field. With the combined effect of the transverse magnetic field from the steering coils and the longitudinal focusing magnetic field from the lens, the electron beam starts a cyclotron motion. In the experiment, the coils are arranged almost parallel to the MTM plates, so they create a transverse magnetic field pointing perpendicular to the plates. With this kicking field, the electron beam with an initial longitudinal velocity gains an initial transverse velocity parallel to the plates, and then the cyclotron motion is initiated. The guiding center of the beam then directs closer to one of the plates. An illustration from a CST simulation of the steered beam profile with 0.1 A of current applied in the steering coil is shown in Fig. 14. The integral of the transverse magnetic field of the pair of steering coils along the beam axis is 182 G-cm/A, and if they were placed in free space, a 490 kV beam would be deflected by 13 milliradians with 0.2 A of current applied in the coils.

Fig. 15 shows a sample of the improved pulse with the steering coils applied. The power pulse now has a better flattop and a higher total power of 2.9 MW, at an efficiency of about 9%. This pulse falls into the second category where the two arms have a frequency difference of 30 MHz.
Fig. 14 Steered beam profile with 0.1 A in the steering coil. The beam energy is 490 kV, the lens field is 725 G, and the solenoid field is 339 G.

Fig. 15 Sample pulse with 0.2 A of current applied in the steering coils. (a) Gun voltage and collector current. (b) Output microwave power. A peak power of 2.9 MW was reached.

The steering coils have also changed the high power operating space. Fig. 16 (a) shows the variation of the high power region (the shaded area) with the applied steering coil current. The beam voltage and the lens field are both fixed. Positive steering current means that the beam is steered closer to Arm 2, and negative current pushes the beam closer to Arm 1. The plot shows the anisotropic feature of the MTM-R structure with reverse symmetry.

Fig. 16 High power operation space with the steering coils applied. (a) High power region as the shaded area. (b) Measurements of the starting voltage with varied steering coil current applied. All the data were taken with a fixed beam voltage of 450 kV and a fixed lens field of 760 G.
The critical voltage for achieving high power (> 1 MW) with different steering coil current values is measured and shown in Fig. 16 (b). In agreement with Fig. 16 (a), a coil current of 0.1 A raises the critical voltage, while a current of -0.2 A lowers the voltage. And when the coil current goes up to 0.6 A, a second valley appears around 650 G.

The measurement with the steering coils on the one hand improves the pulse shape and raises the power, on the other hand, it shows the anisotropic effect of the reverse symmetry in the MTM-R design.

IV. CONCLUSIONS

In conclusion, the MTM-R structure has provided a rich environment to study different types of beam-wave interaction. MTM plates arranged with reverse symmetry operate in a deflecting mode with a negative group velocity to generate high power. PIC simulations show a helical beam trajectory, indicating the Cherenkov-cyclotron instability. In the experiment, megawatt level output power with a full 1 microsecond pulse length was observed with various combinations of the three major parameters: the beam voltage, the lens field and the solenoid field. Coherent radiation was generated at the Cherenkov-cyclotron frequency. A typical total output power for the beam with a voltage of up to 490 kV and current 84 A is 2 to 3 MW with an efficiency of up to 10%. Further investigation of the frequency tuning with the beam voltage, the lens field and the solenoid field identifies different types of interaction at varied operating conditions. Finally, a pair of steering coils were put on to improve the output power traces, and they also reveal the anisotropic feature of the MTM-R structure.

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