Computational studies on scattering of radio frequency waves by density filaments in fusion plasmas

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Abstract

In modern magnetic fusion devices, such as tokamaks and stellarators, radio frequency (RF) waves are commonly used for plasma heating and current profile control, as well as for certain diagnostics. The frequencies of the RF waves range from ion cyclotron frequency to the electron cyclotron frequency. The RF waves are launched from structures, like waveguides and current straps, placed near the wall in a very low density, tenuous plasma region of a fusion device. The RF electromagnetic fields have to propagate through this scrape-off layer before coupling power into the core of the plasma. The scrape-off layer is characterized by turbulent plasmas fluctuations and by blobs and filaments. The variations in the edge density due to these fluctuations and filaments can affect the propagation characteristics of the RF waves – changes in density leading to regions with differing plasma permittivity. Analytical full-wave theories have shown that scattering by blobs and filaments can alter the RF power flow into the core of the plasma in a variety of ways, such as through reflection, refraction, diffraction, and shadowing [see, for example, A. K. Ram and K. Hizanidis, *Phys. Plasmas* **23**, 022504-1–022504-17 (2016) and references therein]. There are changes in the wave vectors and the distribution of power – scattering leading to coupling of the incident RF wave to other plasma waves, side-scattering, surface waves, and fragmentation of the Poynting flux in the direction towards the core. However, these theoretical models are somewhat idealized. In particular, it is assumed that there is step-function discontinuity in the density between the plasma inside the filament and the background plasma. In this paper, results from numerical simulations of RF scattering by filaments using a commercial full-wave code are described. The filaments are taken to be cylindrical with the axis of the cylinder aligned along the direction of the ambient magnetic field. The plasma inside and outside the filament is assumed to be cold. There are three primary objectives of these studies. The first objective is to validate the numerical simulations by comparing with the analytical results for the same plasma description – a step-function discontinuity in density. A detailed comparison of the Poynting flux shows that the numerical simulations lead to the same results as those from the theoretical model. The second objective is to extend the simulations to take into account a smooth transition in density from the background plasma to the interior of the filament. The ensuing comparison shows that the deviations from the results of the theoretical model are quite small. The third objective is to consider the scattering process for situations well beyond a reasonable theoretical analysis. This includes scattering off multiple filaments with different densities and sizes. Simulations for these
complex arrangement of filaments show that, in spite of the obvious limitations, the essential physics of RF scattering is captured by the analytical theory for a single filament.

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I. INTRODUCTION

Radio frequency (RF) waves are commonly used for heating the core of magnetically confined fusion plasmas and for generating currents that aid in the stability of the plasma. The RF waves are launched using structures placed in the vacuum region near the walls. These waves propagate through the tenuous plasma in the scrape-off layer and the edge region before reaching the core and interacting with electrons and ions. The scrape-off layer and the edge region is replete with coherent and turbulent density fluctuations. The former are in the form of blobs and filaments that are extended along the magnetic field line, and spatially confined across it [1–4]; while the latter are distributed randomly in space [5]. Even though the effect on RF waves propagating through these fluctuations has not been quantified experimentally, there is enough indirect evidence that seems to warrant a theoretical and computational study on this topic. The physical reason that one would expect the fluctuations to affect the propagation characteristics of RF waves is quite straightforward. The plasma permittivity inside, and due to, the fluctuations is different from the background permittivity primarily due to modifications in density. Changes in permittivity affect the propagation of RF waves just as changes in the dielectric properties of media affect the propagation of electromagnetic waves – e.g., propagation of light through the air-water interface.

The scattering of plasma waves by spherical blobs and by cylindrical filaments aligned along the magnetic field line has been studied theoretically for conditions that are amenable to a complete analytical formulation [6–8]. These studies show that the RF waves are reflected, refracted, diffracted and side-scattered as they propagate through blobs and filaments. Also, there is shadowing in the wake of the blob or filament. An important finding of these studies was that the incident RF wave can couple power to the other cold plasma wave. This is purely due to the scattering process and not due to any parametric process or mode conversion process. It is a necessary requirement in order to satisfy the electromagnetic boundary conditions at the interface separating the blob or the filament from the background plasma.

This paper discusses in detail results from numerical simulations that are performed to demonstrate the effect of filaments on the propagation of RF waves. Section II outlines the scattering geometry, the wave equation, and the boundary conditions that are the basis
of the theoretical model for scattering off a single cylindrical filament. The details are in [8]. Section III is a brief description of the equations and the boundary conditions that are prescribed in the COMSOL Multiphysics software [9]. In Section IV the simulation code is validated by comparing results obtained from the code with those from the theory. This is done for electron cyclotron waves and lower-hybrid waves. The theory imposes a step-function change in density inside the filament relative to the background plasma. The same condition is set up in COMSOL so that it is possible to directly compare simulation results with those from the theoretical analysis. In Section V, the transition from the background plasma to the inside of the filament is modeled by a smooth function. The computational results for a smooth transition in density are compared with the results for the step-function transition in Section IV. This comparison shows that the analytical model captures all the essential physics of the scattering process. The results obtained from COMSOL for multiple filaments are displayed and discussed in Section VI.

II. THE ANALYTICAL FULL-WAVE MODEL OF SCATTERING

The model for the scattering of RF waves by a cylindrical density filament has been discussed in detail in [8]. What follows is an outline of the essential features of the theory that underlies the scattering model. The results generated using this model will form a basis for comparison with results from the numerical simulations using COMSOL Multiphysics Modeling Software.

A cylindrical filament, having a circular cross-section, is assumed to be infinitely long in the axial $z$-direction. The approximation is valid provided that the width of the RF beam along the axial direction is smaller than the length of the filament. The ambient magnetic field, in which the background plasma and the filament are immersed, is taken to be uniform and also along the $z$-direction (Fig. 1). The plasma inside and outside the filament is assumed to be a cold fluid, and is described by the continuity and momentum equations for the electrons and ions [10]. The electromagnetic fields of waves in the plasma are given by the Faraday’s and Ampere’s equations in Maxwell’s equations. The system of equations is linearized by assuming that the unperturbed plasma state, in the absence of RF waves, is homogeneous in space and stationary in time, both inside and outside the filament. However, the densities in the two regions are allowed to be arbitrarily different.
leading to a discontinuous change in density across the boundary separating the filament from the background plasma.

The perturbed state is assumed to have a plasma density and RF electromagnetic fields that are varying in space and time. For a time variation of the form $e^{-i\omega t}$, where $\omega$ is the angular frequency and $t$ is the time, the propagation of RF waves in a cold plasma is obtained by combining the Faraday’s and Ampere’s equations,

$$\nabla \times (\nabla \times \mathbf{E}(\mathbf{r})) - \frac{\omega^2}{c^2} \mathbf{K}(\mathbf{r}).\mathbf{E}(\mathbf{r}) = 0,$$

where $c$ is the speed of light, and $\mathbf{K}(\mathbf{r})$ is the plasma permittivity tensor. In the cylindrical coordinate system where the ambient magnetic field $\mathbf{B}_0 = B_0 \hat{z}$ is aligned along the $z$-axis, $\mathbf{K}(\mathbf{r})$ has the form [10]

$$\mathbf{K} = \begin{pmatrix} K_\rho & -iK_\phi & 0 \\ iK_\phi & K_\rho & 0 \\ 0 & 0 & K_z \end{pmatrix},$$

where

$$K_\rho = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} - \sum_i \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2},$$

$$K_\phi = -\frac{\omega_{ce}}{\omega} \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} + \sum_i \frac{\omega_{ci}}{\omega} \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2},$$

$$K_z = 1 - \frac{\omega_{pe}^2}{\omega^2} - \sum_i \frac{\omega_{pi}^2}{\omega^2},$$

$\omega_{pe}$ ($\omega_{pi}$) and $\omega_{ce}$ ($\omega_{ci}$) are the angular electron (ion) plasma frequency and cyclotron frequency, respectively, and the index $i$ represents all the ion species in the plasma. The plasma and cyclotron frequencies can, in general, be functions of space. The permittivity tensor of the background plasma and of the filament are expressed in terms of their respective ion compositions and constant, but different, densities. Subsequently, the elements of $\mathbf{K}$ are constants in each region.

The electromagnetic wave equation (1) has the form of a vector Helmholtz equation. It is solved in the cylindrical coordinate system using the vector cylinder functions [8]. Since the density is constant for the background plasma and for the plasma inside the filament,
and since the magnetic field is uniform throughout, (1) along with (2) are solved in each domain separately. The self-consistent solution for the scattered fields and the fields inside the filament is obtained by matching boundary conditions at the interface separating the filament plasma from the background plasma. Assuming that there are no free charges and currents in the interface between the filament and the background plasma, Maxwell’s equations impose the following conditions [11],

\[ \hat{\rho}. (D_I + D_S) \bigg|_{\rho=a} = \hat{\rho}. D_F \bigg|_{\rho=a}, \]  
\[ \hat{\rho}. (B_I + B_S) \bigg|_{\rho=a} = \hat{\rho}. B_F \bigg|_{\rho=a}, \]  
\[ \hat{\rho} \times (E_I + E_S) \bigg|_{\rho=a} = \hat{\rho} \times E_F \bigg|_{\rho=a}, \]  
\[ \hat{\rho} \times (B_I + B_S) \bigg|_{\rho=a} = \hat{\rho} \times B_F \bigg|_{\rho=a}. \]

where \( \hat{\rho} \) is the unit vector along the radial direction in the cylindrical coordinate system. The subscripts \( I, S, \) and \( F \) refer to the incident, scattered, and filamentary wave fields, respectively, \( D = \epsilon_0 \hat{\mathbf{K}}. \mathbf{E} \) is the wave electric displacement field, \( \epsilon_0 \) is the free-space permeability, and \( \mathbf{E} \) and \( \mathbf{B} \) are the wave electric and magnetic field, respectively. The four sets of boundary conditions follow from Gauss’ law, Gauss’ magnetism law, Faraday’s law, and Ampere’s law, respectively. The left and right sides of Eqs. (4) – (7) are evaluated at the boundary of the filament \( \rho = a \). Of the six boundary conditions that comprise (4) – (7), it can be shown that, in a cold plasma, only four are independent [8].

The analytic treatment of the scattering is carried out as follows. A plane wave, propagating in a background plasma with a prescribed frequency and \( k_z \) – the component of the wave vector along the magnetic field line – is incident on the cylindrical filament. Since the axis of the filament is along the magnetic field line, \( k_z \) remains the same for the fields inside the filament and the scattered fields [8]. The scattered wave fields are restricted to be outgoing waves; i.e., their radial group velocity is in a direction away from the filament. In case the scattered fields are not propagating waves, they are restricted to spatially decaying fields. Inside the filament, no such restrictions are imposed. With these assumptions, the electromagnetic fields over all space are uniquely determined for a given amplitude of the electric field of the incident plane wave.
A. The dispersion characteristics and polarization of the plasma waves

The Faraday-Ampere equation (4) is solved using the Fourier transform technique in which the electric field of the RF waves is expressed in terms of plane waves. The waves, whether they are for the incident and scattered fields outside the filament or for the fields inside the filament, have to obey the local dispersion relation [8],

\[
\text{det} \left( \mathbf{D}(\mathbf{k}, \omega) \right) = 0, \tag{8}
\]

where \( \text{det} \) denotes the determinant of the dispersion tensor \( \mathbf{D} \),

\[
\mathbf{D}(\mathbf{k}, \omega) = \frac{c^2}{\omega^2} \left( \mathbf{kk} - k^2 \mathbf{I} \right) + \mathbf{K}. \tag{9}
\]

In (8), \( \mathbf{k} \) is the wave vector, \( \mathbf{kk} \) is a dyadic, and \( \mathbf{I} \) is the unit tensor. In the cylindrical coordinate representation, \( \mathbf{k} \) is expressed as \( \mathbf{k} = (k_\rho, k_\phi, k_z) \). Since the background plasma and the filament are assumed to be azimuthally symmetric, we can set, without any loss of generality, \( k_\phi = 0 \).

In experiments, \( k_z \) is determined by the antenna structure exciting the RF fields. Furthermore, \( k_z \) is preserved in the scattering process. Then equation (8) can be expressed as an algebraic equation for \( k_\rho \) as a function of \( k_z \) and \( \omega \). If we define the index of refraction \( n = c \mathbf{k}/\omega \) with the components \((n_\rho, 0, n_z)\), (8) leads to,

\[
n_\rho^4 K_\rho + n_\rho^2 \left[ K_\phi^2 - (K_\rho + K_z) (K_\rho - n_z^2) \right] + K_z \left[ (K_\rho - n_z^2)^2 - K_\phi^2 \right] = 0. \tag{10}
\]

There are two solutions of the bi-quadratic equation,

\[
n_{\rho \pm} = \frac{1}{2K_\rho} \left( K_\rho + K_z \right) \left( K_\rho - n_z^2 \right) \pm \frac{1}{2K_\rho} K_\phi^2 \sqrt{ \left\{ (K_\rho - K_z) (K_\rho - n_z^2) - K_\phi^2 \right\}^2 + 4n_z^2 K_\phi^2 K_z}. \tag{11}
\]

The four roots of \( n_\rho \) can be easily obtained from (11). If the expression under the radical is greater than zero, there are two distinct values of \( n_\rho^2 \). If the expression under the radical is zero, the two solutions are degenerate, and if it is negative the two roots are a complex conjugate pair.
The two roots $n_{p \pm}^2$ can be classified as follows. For waves propagating across the magnetic field $n_z = 0$. Then, (11) yields,
\begin{align}
  n_{p \pm}^2 &= \frac{K_p^2 - K_\phi^2}{K_\rho}, \quad (12) \\
  n_{p -}^2 &= K_z. \quad (13)
\end{align}

In the electron cyclotron range of frequencies, (12) and (13) describe the extraordinary and ordinary waves of propagation, respectively. In the lower-hybrid range of frequencies they correspond to fast and slow lower-hybrid waves, respectively. While in the ion cyclotron range of frequencies they are the fast Alfvén wave and slow Alfvén wave, respectively. This nomenclature carries over for $n_z \neq 0$. The theoretical model and analysis are valid for all these waves.

The polarization of the waves is obtained from,
\begin{equation}
  \mathbf{D} \mathbf{E} = 0, \quad (14)
\end{equation}

B. The Poynting vector

Some of the features that result from the scattering model are best displayed by the time-averaged Poynting vector [12],
\begin{equation}
  \langle S(t) \rangle = \frac{1}{2} \text{Re} (\mathbf{E} \times \mathbf{H}^*), \quad (15)
\end{equation}

where $\langle \ldots \rangle$ denotes the time average, and $\mathbf{H}^*$ is the complex conjugate of the magnetic intensity $\mathbf{H}$. In a plasma, the wave magnetic field $\mathbf{B}$ is related to $\mathbf{H}$ by $\mathbf{B} = \mu_0 \mathbf{H}$, where $\mu_0$ is the free-space permeability. Since all the electromagnetic fields are proportional to the amplitude of the electric field of the incident plane wave, it is more appropriate to quantify the normalized Poynting vector
\begin{equation}
  P = \frac{\langle S(t) \rangle}{|\langle S_I \rangle|}, \quad (16)
\end{equation}

where $|\langle S_I \rangle|$ is the magnitude of the time-averaged Poynting vector of the incident plane wave.
III. THE COMPUTATIONAL MODEL

The COMSOL Multiphysics Modeling Software is used to solve Maxwell’s equations in a bounded two-dimensional domain, which is part of a plane perpendicular to the magnetic field along $\hat{z}$. The plasma permittivity in this domain is that of a cold plasma, given by (2), and the plasma density can be prescribed in any arbitrary sub-domain of the computational domain. There is no limitation on the number and shape of the sub-domains with the only assumption being that the domain and the sub-domains are infinitely extended along $z$. Unlike the theoretical model, no boundary conditions are imposed within the domain – the software automatically ensures that the electromagnetic fields in the interior of the domain satisfy Maxwell’s equation. However, conditions on the wave fields have to be imposed at the boundary of the domain. This is accomplished by surrounding the computational domain with a Perfectly Matched Layer (PML) at the boundary, which is properly configured for the prescribed incoming electromagnetic plane wave. The PML boundary ensures that all outgoing waves essentially “leave” the computational domain. Just as in the theoretical model, for the incoming plasma wave $k_z$ and $\omega$ are specified a priori, and the component of the wave vector normal to the magnetic field is chosen to be one of the roots of the plasma dispersion relation for the background plasma. The simulations can be done in a two-dimensional domain as $k_z$ does not change due to scattering. Effectively, the domain in the $z$-direction is periodic with period $2\pi/k_z$.

IV. COMPARISON OF ANALYTICAL AND SIMULATION RESULTS – STEP-FUNCTION DENSITY PROFILE

In order to validate the simulation code, we compare results from the theoretical model with those obtained using COMSOL. We choose plasma parameters that one may encounter in a high density and high magnetic field fusion device. Besides comparing the two approaches, the idea is to illustrate the physics of the scattering process without being too particular about the exact plasma parameters in the scrape-off layer.
A. Electron cyclotron waves

In an ambient magnetic field $B_0 = 4.5$ T, the incident electron cyclotron plane wave with frequency $f_0 = 170$ GHz is taken to propagate in a direction perpendicular to the magnetic field, i.e., $n_z = 0$. We assume that the electron plasma density inside the filament is $n_{ef} = 1.5 \times 10^{19}$ m$^{-3}$, while the background electron plasma density is $n_{eb} = 10^{19}$ m$^{-3}$. The plasma ions are assumed to be deuterons.

In the first set of results we consider an extraordinary plane wave incident on a cylindrical filament of radius $a = 1$ cm. The wave is assumed to be propagating along the $x-$ direction, i.e., for the incident wave $\hat{\rho} = \hat{x}$. Consequently, $n_z = 0$ for all the waves. From the dispersion relation for the background plasma, we obtain $n_{\rho X} \approx 0.967$ for the incident extraordinary wave, and $n_{\rho O} \approx 0.986$ for the ordinary wave. The corresponding wavelengths are, approximately, 0.182 cm and 0.179 cm, respectively. Inside the filament, the radial wave numbers are $n_{\rho X}^f \approx 0.95$ for the extraordinary wave, and $n_{\rho O}^f \approx 0.979$ for the ordinary wave. The corresponding wavelengths are, approximately, 0.186 cm and 0.18 cm, respectively. Thus, all the modes, inside and outside the filament, are propagating waves having wavelengths that are much shorter than the radius of the filament.

For the incident extraordinary wave, the Cartesian $(\hat{x}, \hat{y}, \hat{z})$ components of the incident wave Poynting vector are $\langle \mathbf{S}_{IX} \rangle \approx (0.965, 0, 0)$, respectively, with the assumption that the magnitude of the electric field for the incoming wave is 1. The normalized Poynting vector for the incoming wave is $\mathbf{P}_{IX} = (1, 0, 0)$.

Figures 2a and 2b show, respectively, contour plots for $P_x$ and $P_y$, the $x$- and $y$ components of the normalized Poynting vector, in the $x-y$ plane, as obtained from the theoretical model. In each figure, the white circle marks the boundary of the filament and the wave is incident from the left hand side. Both figures show that the extraordinary wave is scattered in the forward direction and there is essentially no backscattering due to the filament. In the forward direction, the scattering leads to diffraction patterns with a spatially corrugated power flow. Just behind the filament is the shadow region with a reduced Poynting flux. Figure 2a is the power flow towards the core of the plasma, while Fig. 2b shows side-scattering of the incident wave. Since the $y$-component of the Poynting vector for the incident wave is zero, the non-zero values seen in Fig. 2b are due to the scattering off the filament. The scattering is symmetric in the $\pm y$-directions as the incident wave has $n_z = 0$. 

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The \( z \)-component of the Poynting vector \( P_z \) is zero for the incoming wave and remains zero for the scattered waves. While the power flow is spatially corrugated in the plasma outside the filament, the power flow in the interior of the filament is akin to a cavity mode with a tendency of focusing in the forward direction.

Figures 3a and 3b show the simulation results obtained using COMSOL. A comparison with Figs. 2a and 2b, respectively, shows excellent agreement between the theoretical and simulation results. \( P_z \) is zero just as it is in the analytical results.

The next few figures provide further insight into the scattering process, as well as a more rigorous comparison between theory and simulations. Figures 4a and 4b are line plots for \( P_x \) and \( P_y \), respectively, as a function of \( y \) for fixed \( x \). These results follow from Figs. 2a-3b. The solid (blue) and the dashed (red) lines are obtained from theory and simulations, respectively, at \( x = 1.5 \) cm located in the region of forward scattering. The small dashed line (black) in Figs. 4a and 4b are for \( x = -1.5 \) cm in the back-scattering region – there is essentially no back-scattering. The agreement between theory and computations is quite obvious.

The Poynting flux in the forward direction is spatially structured. For the incident wave, the \( x \)-component of the normalized Poynting vector \( P_{Ix} \) is 1. In Fig. 4a, regions where \( P_x > 1 \) are indicative of constructive interference between the incident and scattered waves, akin to a focusing effect due to the filament. Meanwhile, the \( P_x < 1 \) are regions of destructive interference and lead to the shadowing phenomenon. Figure 4b shows that, in the wake of the filament, \( P_y \neq 0 \). This implies side-scattering of the incident wave by the filament, since the incoming wave has \( P_{Iy} = 0 \). As the initial direction of propagation is along the \( x \)-axis, side-scattering is symmetric around \( y = 0 \). Thus, scattering is prominent in the wake of the filament, while it is essentially negligible in front of the filament – indicating little or no reflection.

The ordinary wave scatters differently from the extraordinary wave. For the parameters mentioned above, the radial index of refraction for the ordinary wave is \( n_{\rho O} \approx 0.986 \). The Cartesian components of Poynting vector for the incident ordinary wave are \( \langle \mathbf{S}_{IO} \rangle \approx (0.986, 0, 0) \), which corresponds to the normalized Poynting vector with components \( \mathbf{P}_{IO} \approx (1, 0, 0) \).

Figures 5a and 5b display \( P_x \) and \( P_y \), respectively, as a function of \( y \) in the forward scattering region at \( x = 1.5 \) cm. In these results from COMSOL simulations, the solid
(blue) line is for an incident extraordinary wave and the dashed (red) line is for an incident ordinary wave. The spatial variation in $P_x$ and $P_y$ is reduced for the ordinary wave. However, the peaks and valleys in $P_x$ and $P_y$, within the primary scattering region, occur essentially at the same location in $y$.

The results shown in Figs. 6a and 6b are similar to Figs. 5a and 5b, respectively, except that the radius of the filament is chosen to be $a = 2$ cm and the variation, as a function of $y$, is evaluated at $x = 2.5$ cm, just outside the filament. This is in the region of forward scattering and the figures compare the scattering of an incident extraordinary wave (solid line) with that of an ordinary wave (dashed line). Again, the spatial variation for the extraordinary wave is over a larger range than for the ordinary wave. Also, comparing with Figs. 5a and 5b, the variation in the Poynting vector for both incident waves is relatively larger when scattering off a filament with a larger radius. There is an increase in the side-scattered power as the filament radius is increased. The displayed results are obtained from the theoretical model.

**B. Lower-hybrid waves**

The frequency of the incident RF wave is taken to be $f_0 = 4.6$ GHz and we set $n_z = 2$, so that the parallel wavelength for all waves is 3.26 cm. This wavelength is bigger than the radius of the filament, which is taken to be 1 cm. The plasma composition and the densities of the background plasma and that inside the filament are taken to be the same as for the electron cyclotron waves. Then, the two roots obtained from (11) are $n_{\rho+}^2 \approx -2.359$ and $n_{\rho-}^2 \approx 103.081$, which correspond to the “fast wave” and slow wave, respectively. The “fast wave” is not a propagating wave as $n_{\rho+}$ is imaginary. For the incident slow wave we choose $n_{\rho-} \approx -10.153$ – the corresponding wavelength is 0.642 cm, which is comparable to the radius of the filament. The slow wave is a backward wave [10] – the group velocity is in a direction opposite to the phase velocity. Thus, for a wave propagating with its $x$-component of the Poynting vector along the $+x$ direction, $n_{\rho-}$ has to be negative. The Cartesian components of the incident wave Poynting vector are $\langle S_I \rangle \approx (0.073, 0, 0.519)$ with the amplitude of the wave electric field set to 1. The normalized Poynting vector for the incoming wave is $P_I = (0.14, 0, 0.99)$. Inside the filament, for the slow wave, $n_{\rho-}^f \approx \pm 12.25$, corresponding to a wavelength of 0.532 cm.
Figures 7a, 7b, and 7c show the variation of $P_x$, $P_y$, and $P_z$, respectively, in the $x−y$ plane due to scattering by a filament of radius 1 cm. Since the wave is incident from the left, the figures show the patterns that emerge from the interference between the incident wave and the scattered wave in the forward direction and in the wake of the filament. Unlike the scattering of electron cyclotron waves, the back-scattering of the slow lower-hybrid wave is quite significant, while the forward scattering is much less affected by the filament. The pattern of spatial corrugations in the background plasma, especially on the side of filament facing the incoming wave, is due to the backward nature of the slow lower-hybrid wave. The significant side-scattering of the wave, as shown in Fig. 7b, is not symmetric about the $x$–axis as the incident wave is propagating obliquely with respect to the direction of the magnetic field. In contrast to the case of electron cyclotron waves, Fig. 7c shows interference patterns inside the filament and power flow guided along the axis. These are like cavity or waveguide modes. The results in Figs. 7a, 7b, and 7c are obtained using the COMSOL simulation package and are in excellent agreement with the results obtained from the theoretical model.

The line plots in Figs. 8a, 8b, and 8c illustrate the physics of the scattering of slow lower-hybrid waves. The plots show the three Cartesian components of the Poynting vector as a function of $y$ at two different locations in $x$–these are obtained from the contour plots in Figs. 7a, 7b, and 7c. In each plot the dashed line is at $x = −1.5$ cm, while the solid line is at $x = 1.5$ cm. Figures 8a and 8c show small changes in $P_x$ and $P_z$, respectively, in front of the filament, while there is considerable variation in the wake of the filament. Near $y \approx ±1$ some of the power is reflected back in the negative $x$-direction. Since the incident wave $P_{iy} = 0$, Fig. 8b shows that there substantial side-scattering of the wave in the front and in the wake of the filament. The front of the filament is defined as the side facing the incoming wave, i.e., the side for which the outward pointing normal is opposite to the direction of the group velocity of the wave. The wake of the filament is defined as the side away from the incoming wave, where the outward normal is in the same direction as the group velocity of the wave.

For a filament with a larger radius, the scattering effects as for the electron cyclotron waves. Figures 9a, 9b, and 9c show the three Cartesian components of the Poynting vector for a filament with $a = 2$ cm. Comparing these with Figs. 8a, 8b, and 8c, respectively, we note that the range of variation of the Poynting vector, as a function of $y$, is approximately
V. SIMULATION RESULTS FOR A CONTINUOUS DENSITY PROFILE

The analytical formulation has been for a discontinuous transition in the plasma density from inside the filament to the outside background plasma. It is a step-function discontinuity at the boundary between the uniform density inside the filament and the uniform density outside. All the numerical results discussed above, whether they were from our theory or from COMSOL simulations, are for such a density profile. However, one would normally expect a smooth transition in density from the background plasma to inside the filament. While it is difficult to formulate an analytical solution to the scattering process for a continuous density profile, the same is not true when using the COMSOL simulation software.

For a filament of radius $a$, we assume a continuous transition in density in the region $a \leq r \leq a + dr$. Inside the filament for $r \leq a$, the electron density is $n_{ef}$. The background electron density for $r \geq a + dr$ is $n_{eb}$. Then, in the COMSOL simulations, we assume the following density profile in the transition region,

$$n_e(r) = \frac{n_{ef} + n_{eb}}{2} - \frac{n_{ef} - n_{eb}}{2} \cos \left\{ \pi \left( \frac{(a + dr) - r}{dr} \right) \right\}, \quad \text{for } a \leq r \leq a + dr. \quad (17)$$

The density profile is illustrated in Fig. 10.

A. Electron cyclotron waves

We assume that the wave parameters are the same as in Section IV A. Figure 11a compares $P_x$ as a function of $y$ for $dr = 0.05$ cm (dashed line) with that for $dr = 0.25$ cm (solid line) as obtained from COMSOL simulations. Here $a = 1$ cm and the incident wave is the extraordinary wave. The result for $dr = 0.05$ cm is the same as for the step-function density profile (Fig. 5a). This shows that the step-function profile used in the analytical solution is a very good approximation. The other components of the Poynting vector are in similar agreement.

Figure 11b is for an ordinary wave incident on a filament with $a = 2$ cm. Here the difference in $P_x$ as a function of $y$ between the two density profiles is slightly more that

the same for the two different radii.
in Fig. 11a. But, this still shows that the analytical solution (Fig. 6a), which compares favorably with \( dr = 0.05 \) cm, describes the scattering process quite accurately. In the region in front of the filament, the back-scattered Poynting vector for the step-function density profile is the same as for the continuous profiles with different \( dr \).

**B. Lower-hybrid waves**

The results discussed in this section are for wave parameters that are in Section IV B. The slow lower-hybrid wave incident is assumed to be incident on a filament with \( a = 2 \) cm. Figure 12a shows \( P_x \) versus \( y \) in the front of the filament at \( x = -2.5 \) cm. The dashed (blue) line is for the step-function density profile, while the solid (red) line is for the continuous density profile with \( dr = 0.25 \) cm. Figure 12b shows the results in the wake of the filament at \( x = 2.5 \) cm. Figures 13a and 13b are similar plots for \( P_y \) at \( x = -2.5 \) cm and \( x = 2.5 \) cm, respectively, while 14a and 14b are plots for \( P_z \). From these figures for the various Cartesian components of the Poynting vector, we note that the analytical results for a step-function profile are in good agreement with those for a continuous density profile.

**VI. SCATTERING OFF MULTIPLE FILAMENTS**

In the scrape-off layer and in the edge region of a tokamak plasma, it is more likely to have multiple filaments than a single filament. It is highly onerous, if not impossible, to seek an analytical solution to the scattering of RF waves by two or more filaments of arbitrary size and densities. This is more effectively and efficiently handled using the COMSOL simulation package. The results for a single filament are instructive as they provide a physical insight into the scattering process – even for multiple filaments.

For illustrative purposes, we consider wave scattering off five filaments with differing densities and radii. In Table I we list the location of the center of each filament in the \( x - y \) plane, its radius, and the density. The background density is taken to be \( n_{eb} = 10^{19} \) m\(^{-3}\) as in Section V.
TABLE I: Location, radius, and density of each filament

<table>
<thead>
<tr>
<th>filament</th>
<th>center $(x, y)$ (cm)</th>
<th>radius (cm)</th>
<th>density (m$^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$(-3, -3)$</td>
<td>1</td>
<td>$1.5 \times 10^{19}$</td>
</tr>
<tr>
<td>2</td>
<td>$(-0.5, -1.2)$</td>
<td>2</td>
<td>$1.3 \times 10^{19}$</td>
</tr>
<tr>
<td>3</td>
<td>$(-2, 1)$</td>
<td>0.5</td>
<td>$0.85 \times 10^{19}$</td>
</tr>
<tr>
<td>4</td>
<td>$(2, 2)$</td>
<td>0.75</td>
<td>$0.5 \times 10^{19}$</td>
</tr>
<tr>
<td>5</td>
<td>$(3.5, -1.5)$</td>
<td>1.3</td>
<td>$1.8 \times 10^{19}$</td>
</tr>
</tbody>
</table>

A. Electron cyclotron waves

For the parameters stated in Section IV A, consider an incident ordinary wave. Figures 15 and 15b show contours of $P_x$ and $P_y$, respectively, in the $x-y$ plane. For $y < 0$, the scattering is dominated by the lead filament with its center located at $(x, y) = (-3, -3)$. The filaments that are in the wake do not seem to affect the scattering pattern. Line plots of $P_x$ and $P_y$ versus $y$ displayed in Figs. 15b and 16a, respectively, show that the dominant scattering is in the forward direction in the wake of the filaments. The back scattering is essentially negligible. When comparing with the ordinary wave results for a single filament shown in Figs. 5a and 5b, we note that the variation in the components of the Poynting vector is over a larger range for multiple filaments. Also, the side-scattering is enhanced.

B. Lower-hybrid waves

Figures 16b, 17a, and 17b show contours of $P_x$, $P_y$, and $P_z$, respectively, in the $x-y$ plane for parameters given in Section IV B. In contrast to the results for the ordinary electron cyclotron wave, the scattering is influenced more significantly by filaments in the back, with centers located in the region $x > 0$. The line plots of $P_x$, $P_y$, and $P_z$ in Figs. 17c, 18a, and 18b, respectively, show that the variation in the components of the Poynting vector are over a larger range than for a single filament (see Figs. 9a, 9b, and 9c).
VII. CONCLUSION

There are two sets of primary conclusions that can drawn from the results discussed in this paper.

The first set of conclusions is related to the validation, and extension, of the computations carried out within the framework of the COMSOL modeling software. COMSOL has been set up to study the scattering of radio frequency waves by filamentary structures in a cold magnetized plasma. An important aspect in developing the computational code is the implementation of boundary conditions at the edges of the simulation region. The incoming plane wave is imposed at one boundary of the simulation region. The scattered waves are required to be outgoing waves with no reflections at the boundary. Otherwise, the results are completely erroneous. In order to ensure proper implementation of the boundary conditions, the code has been validated against analytical results that have been rigorously derived from Maxwell’s equations [8]. There is excellent agreement between results from the simulations with those from the analytical formalism. This gives us confidence when using the COMSOL code for studying scattering in situations which are beyond the scope of the theoretical model. Subsequently, the COMSOL simulations have been used to show that the theoretical formalism is valid beyond its restrictive domain for scattering from a single cylindrical filament. The theory is for a step-function discontinuity between the plasma density inside the filament and the density of the background plasma. We have used COMSOL to simulate scattering of waves off a filament when the transition in density is smooth. For different scale lengths of the transition region, the Poynting vector in the scattered region is well approximated by the analytical theory. It is also shown that the COMSOL software can be confidently extended to study scattering off multiple filaments. In fact, it could be used to study scattering off fluctuations with arbitrary spatial profiles and distributions. These sort of scattering situations do not lend themselves to some reasonably feasible theoretical and analytical analysis.

The second set of conclusions has to do with the scattering process itself and the results that follow from various simulations. For a cylindrical filament with its axis aligned along the magnetic field line, the axial component of the wave vector $k_z$ is preserved for the scattered electromagnetic fields and for the fields inside the filament. The scattering leads to the presence of waves in the background plasma that cover the entire azimuthal plane.
perpendicular to the axis. This follows from the requirement that boundary conditions, derived from Maxwell's equations, be satisfied at the interface separating the filament from the background plasmas. Since the background plasma, in the vicinity of the filament, is uniform, and since there is azimuthal symmetry, the radial component of the wave vector of all the scattered waves is the same. The scattered waves and the incident wave interfere constructively and destructively leading to spatially structured fields. Let us define a plane of incidence that is spanned by the wave vector of the incident plane wave and the ambient magnetic field vector. Then, an important consequence of scattering is that some of the incident wave power is side-scattered.

The spatial behavior of the Poynting vector is a suitable way to illustrate the effect of filaments on the propagation characteristics of the radio frequency waves. We have explored two frequency regimes – the electron cyclotron frequency and the lower-hybrid frequency. As discussed in the previous paragraph, regardless of the frequency of the incoming plane wave, the scattering by a filament leads to a spatially non-uniform Poynting vector. However, there is a distinct difference between the two frequency regimes. For electron cyclotron waves, the scattered waves affect the Poynting vector in the wake of the filament; primarily, in the direction determined by the Poynting vector of the incoming wave. The back-scattering is essentially negligible. For lower-hybrid waves, the Poynting vector in front of the filament facing the incoming wave is also affected, indicating significant back-scattering.

For electron cyclotron waves, the extraordinary wave and the ordinary wave are scattered prominently in the forward direction. The extraordinary wave is more affected by the scattering than the ordinary wave – the former having larger deviations of the Cartesian components of the Poynting vector. The deviations change and become somewhat larger as the radius of a filament is increased. In the wake of the filament, there are spatial regions where the magnitude of the Poynting vector is greater than that of the incident wave, and regions where it is less. Both waves are side-scattered so that the power coupled into the core of the plasma is reduced.

The densities chosen for the plasma inside the filament and for the background plasma, only the slow wave propagates in the lower-hybrid frequency range. The slow wave is a backward wave so that the phase fronts formed by the beating of the incident wave and the scattered wave are in a direction opposite to that of the group velocity which is along the direction of the Poynting vector. The lower-hybrid wave is not only scattered in the forward
direction but also in the backward direction. The back-scattering dominates and some of the power is reflected back towards the boundary from where the incoming wave originates. There is side-scattering as well and the spatial pattern of the Poynting vector changes as the radius of the filament is increased. However, there is no substantial change in the range over which the Poynting vector fluctuates as the radius of the filament is increased.

Finally, it is interesting to note that the spatial changes in the magnitude of the Poynting vector for multiple filaments are in the same range as for a single filament. The primary differences is that the spatial scales over which the changes take place is shorter than that for a single filament.

The theoretical and computational analysis of scattering of radio frequency waves by cylindrical filaments, aligned along the magnetic field line, shows that the Poynting flux becomes spatially non-uniform. This lack of uniformity and the associated side-scattering of some of the incident wave power is an indication that the edge turbulence in tokamak plasmas is very likely to affect the propagation characteristics of RF waves. This, in turn, could modify the heating and current drive profiles in the core of a tokamak plasma, as well as reduce the efficiency since some of the wave power gets coupled to surface wave and remains in the edge region.

VIII. ACKNOWLEDGEMENTS

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FIG. 1: (Color online) The origin of the coordinate system is at the center of the cylindrical filament of radius $a$. The ambient magnetic field is along the axial direction $\hat{z}$, and $\hat{\rho}$ and $\hat{\phi}$ are the unit vectors along the radial and azimuthal directions, respectively. The effect of the ends of the filament are ignored by assuming that $L \to \infty$. 
FIG. 2: (Color online) Contours of (a) $P_x$ and (b) $P_y$ in the $x - y$ plane derived from the theoretical model. The electron cyclotron extraordinary wave is incident from the left hand side with its propagation vector along the $x$–direction. The circular cross-section of the filament, centered at $x = y = 0$, has a radius of 1 cm, and is indicated by the white circle. The plasma parameters corresponding to this plot are given in the text. Since the incoming wave has $P_y = 0$, (b) shows side-scattering of the incident extraordinary wave.
FIG. 3: (Color online) This figure shows the contours of (a) $P_x$ and (b) $P_y$ in the $x-y$ plane obtained using the COMSOL Multiphysics Modeling Software. The input plasma parameters are the same as for Fig. 2a. The results from the simulation code agree very well with results from the theoretical model shown in Figs. 2a and 2b.
FIG. 4: (Color online) A comparison between the analytical theory and COMSOL simulations of the Cartesian components of the Poynting vector: (a) $P_x$ and (b) $P_y$. The solid (blue) line is from theory and the dashed (red) line is from simulations. These results are evaluated in the region of forward scattering at $x = 1.5$ cm. $P_y \neq 0$ indicates that the incident wave is side-scattered by the filament. The small dashed lines near $P_x \approx 0$ and $P_y \approx 0$ are in the back-scattering region at $x = -1.5$ cm. In this region the theory and simulation results are essentially the same.
FIG. 5: (Color online) (a) $P_x$ and (b) $P_y$ as a function of $y$ at $x = 1.5$ cm as obtained from the COMSOL simulations. The solid (blue) line is for an incident extraordinary wave, while the dashed (red) line is for an incident ordinary wave. The plasma parameters are as discussed in the text. $P_y \neq 0$ indicates side-scattering of the wave by the filament.

FIG. 6: (Color online) (a) $P_x$ and (b) $P_y$ as a function of $y$ at $x = 2.5$ cm from the theoretical model. As opposed to figures 2a – 5b, these scattering results are for a filament with radius $a = 2$ cm. The solid (blue) line is for an incident extraordinary wave and the dashed (red) line is for an incident ordinary wave.
FIG. 7: (Color online) COMSOL simulations showing contours of (a) $P_x$, (b) $P_y$, and (c) $P_z$, in the $x - y$ plane for an incident slow lower-hybrid wave. The radius of the filament is 1 cm with its center located at $x = y = 0$, and the wave is incident from the left. The other parameters are as discussed in the text. The lower-hybrid wave is a backward wave - its Poynting vector being opposite to its phase velocity. The diffraction patterns are evidence of this backward nature of the wave. Since the incident wave has $P_{ly} = 0$, (b) is indicative of side scattering of the wave.
FIG. 8: (Color online) Line plots for (a) $P_x$, (b) $P_y$, and (c) $P_z$, as a function of $y$. These follow from Figs. 7a, 7b, and 7c, respectively. The solid (blue) line is in the wake of the filament at $x = 1.5$, while the dashed (red) line is in front of the filament at $x = -1.5$ cm.
FIG. 9: (Color online) Line plots for (a) $P_x$, (b) $P_y$, and (c) $P_z$, as a function of $y$ for the slow lower-hybrid wave obtained from COMSOL simulations. These figures differ from Figs. 8a, 8b, and 8c in that the radius of the filament is $a = 2$ cm. The solid (blue) line is in the wake of the filament at $x = 2.5$, while the dashed (red) line is in front of the filament at $x = -2.5$ cm.
FIG. 10: (Color online) Pictorial representation for the smooth density profile. The filament region is for $r \leq a$, the transition region is for $a \leq r \leq a + dr$, and the background plasma is in the region $r \geq a + dr$. The density profile in the transition region is given in (17).
FIG. 11: (Color online) COMSOL simulations comparing the results for two different density profiles for electron cyclotron waves. In both figures the dashed (blue) line is for $dr = 0.05$ cm and the solid (red) line is for $dr = 0.25$ cm. (a) $P_x$ as a function of $y$ at $x = 1.5$ cm in the wake of a filament of radius $a = 1$ cm. The incident wave is an extraordinary wave. The result for $dr = 0.05$ cm is the same as for the step-function density profile given by the solid (blue) line in Fig. 5a. (b) $P_x$ as a function of $y$ at $x = 2.5$ cm in the wake of a filament of radius $a = 3$ cm. The incident wave is an ordinary wave. The result for $dr = 0.05$ cm is the same as for the step-function density profile given by the solid (blue) line in Fig. 5a.
FIG. 12: (Color online) COMSOL simulations comparing the results for two different density profiles for the slow lower-hybrid wave. The filament radius is $a = 2$ cm. The solid (red) line is for density profile with $dr = 0.25$ cm, while the dashed (blue) lines is from the analytical result with a step-function density profile. (a) $P_x$ as a function of $y$ at $x = -2.5$ cm in front of the filament. The dashed (blue) line is the same as the dashed (red) line in Fig. 9a. (b) $P_x$ as a function of $y$ at $x = 2.5$ cm in the wake of the filament. The dashed (blue) line is the same as the solid (blue) line in Fig. 9a.
FIG. 13: (Color online) These figures are for $P_y$ as a function of $y$ for the same situation as in Figs. 12a and 12b, respectively. (a) The dashed (blue) line is the same as the dashed (red) line in Fig. 9b. (b) The dashed (blue) line is the same as the solid (blue) line in Fig. 9b.

FIG. 14: (Color online) These figures are for $P_z$ as a function of $y$ for the same situation as in Figs. 12a and 12b, respectively. (a) The dashed (blue) line is the same as the dashed (red) line in Fig. 9c. (b) The dashed (blue) line is the same as the solid (blue) line in Fig. 9c.
FIG. 15: (Color online) The contour plots of (a) $P_x$ and (b) $P_y$ in the $x - y$ plane for five filaments. The ordinary electron cyclotron wave is incident from the left hand side. The location and radius of each filament, and the electron density inside it are given in Table I.
FIG. 16: (Color online) The Cartesian components of the Poynting vector: (a) $P_x$ versus $y$, and (b) $P_y$ versus $y$, for the multi-filament configuration in Figs. 15 and 15b. In both figures, the solid (blue) is the curve for $x = 4.5$ cm in the wake of the filaments, and the dashed (red) line is for $x = -4.5$ cm in front of the filaments. The forward scattering dominates, while there is little back scattering.
FIG. 17: (Color online) Contour plots of (a) $P_x$, (b) $P_y$, and (c) $P_z$ in the $x−y$ plane for a slow lower-hybrid wave, incident from the left, interacting with five filaments.
FIG. 18: (Color online) The Cartesian components of the Poynting vector: (a) $P_x$, (b) $P_y$, and (c) $P_z$ as a function of $y$. These plots follow from Figs. 17a, 17b, and 17c, respectively. The solid (blue) lines are at $x = 4.5$ cm in the wake of the filaments, and the dashed (red) lines are at $x = -4.5$ cm in front of the filaments.