Scattering of Radio Frequency Waves by Density Fluctuations in Tokamak Plasmas

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Abstract:
In tokamak fusion plasmas, coherent fluctuations in the form of blobs or filaments and incoherent fluctuations due to turbulence are routinely observed in the scrape-off layer. Radio frequency (RF) electromagnetic waves, excited by antenna structures placed near the wall of a tokamak, have to propagate through the scrape-off layer before reaching the core of the plasma. RF waves in the electron cyclotron and lower hybrid range of frequencies are commonly used to modify the current profile. In the International Thermonuclear Experimental Reactor (ITER), electron cyclotron waves are expected to stabilize the neoclassical tearing mode by providing current in the island region. While the effect of fluctuations on RF waves has not been quantified experimentally, there are telltale signs, arising from differences between results from simulations and from experiments, that fluctuations can modify the spectrum of RF waves. Consequently, pioneering theoretical studies and complementary computer simulations have been pursued to elucidate the impact of fluctuations on RF waves. These studies, using the full complement of Maxwell’s equations for a cold, magnetized plasma, show that the Poynting flux in the wake of the filament develops spatial structure due to diffraction and shadowing. The uniformity of power flow into the plasma is affected by side-scattering, modifications to the wave spectrum, and coupling to plasma waves other than the incident RF wave. The Snell’s law and the Fresnel equations have been reformulated within the context of magnetized plasmas. These are distinctly different from their counterparts in scalar dielectric media, and reveal new and important physical insight into the scattering of RF waves. All of these studies apply to the scattering of RF waves in any frequency range and for arbitrary variations in density.

1 Introduction

The scrape-off layer in a tokamak fusion plasma is replete with incoherent and coherent fluctuations – the latter being intermittent and in the form of blobs and filaments \cite{1, 2, 3, 4}.

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The edge plasma plays an important role in the transport of energy and particles to the divertor region and the plasma facing components, as well as in the fusion performance of the core plasma.

Radio frequency waves, commonly used for heating the plasma and for generating non-inductive plasma current, are excited by an external antenna structure at the edge of a fusion device. The RF waves have to propagate through the turbulent scrape-off layer on their way toward the core of the plasma. While the effect of blobs and filaments on RF waves has not been evaluated experimentally, several theoretical studies have been undertaken to, qualitatively and quantitatively, determine the effect of coherent fluctuations on RF waves \[6, 7, 8\]. The physical reason that such a study is relevant is quite straightforward. Since the plasma permittivity inside the filaments is different compared to the background plasma, the characteristic properties of the incident RF waves can change during their transit through the fluctuations. This is clearly the case in the conventional propagation of electromagnetic waves through different dielectric media \[9\]. It is common to study these changes using the geometric optics approximation (see \[10\] and references therein), wherein, rays, representing plane waves, are refracted due to changes in the plasma permittivity. However, the domain of validity of this approximation is quite limited; it requires that \(\delta n \equiv |n_f - n_b|/n_b \ll 1\) \((n_f\) is the density of electrons inside the fluctuation and \(n_b\) is the background density). However, from experimental observations, typically, \(\delta n \in (0.05, 1)\) in the scrape-off layer \[5\]. Consequently, a description of the scattering process requires a more general approach that is not limited by the bounds of the geometric optics approximation.

The approach to a full-wave theoretical model, using Maxwell’s equations, is outlined in this paper. This study is specific to scattering by a cylindrical filament that is aligned along the ambient magnetic field line. The model, based on the conventional Mie theory for scattering of vacuum electromagnetic waves by scalar dielectrics, is appropriate for magnetized plasmas for which the permittivity is given by a tensor.

In order to understand the effect of the filament on RF fields, and to remain consistent with the approximations described below, it is appropriate to study the wave fields in close proximity of the filament. It is assumed that the background plasma around the filament and inside the filament has uniform density and is cold – thermal effects in the scrape-off layer are ignored. However, \(\delta n\) is allowed to be completely arbitrary and not limited to small density fluctuations. The theoretical framework applies to plasma waves of any frequency, including ion cyclotron, lower hybrid, and electron cyclotron waves.

Observations point to filaments having a correlation length of meters along the magnetic field line, and ranging from 0.1 cm to 10 cm perpendicular to the magnetic field \[5\]. The RF beam size, limited by the toroidal extent of the antenna structures, is small compared to the size of the filament along the magnetic field line. It is also small compared to the toroidal circumference of the tokamak. Thus, the toroidal plasma is assumed to have a large aspect ratio; in fact, it is approximated by an infinitely extended slab in the direction of the magnetic field. The cylindrical filament is assumed to be of infinite axial extent along the magnetic field; thereby, effects due to the ends of a filament of finite axial length are ignored.

Beyond their respective domain of validity, there are three primary differences between
the full-wave theory and the geometric optics approximation. First, whereas geometric optics is used for describing refractive changes in the ray propagation, the full-wave model also includes effects due to reflection, diffraction, and shadowing. Second, in geometric optics, the character of the wave does not change as the RF ray propagates through fluctuations. For example, an incident ordinary wave in the electron cyclotron range of frequencies will remain an ordinary wave during its encounter with fluctuations. In the full-wave model, the fluctuations can couple power to other plasma waves. Thus, for the example considered, the ordinary wave couples power to the extraordinary wave. This is not nonlinear parametric coupling; it is linear coupling facilitated by the fluctuations. Third, due to diffraction of waves by the filament, the scattered waves propagate in all radial directions relative to the magnetic field line. Consequently, fluctuations can scatter some of the incident wave power to surface waves which do not propagate into the core plasma. And, for an incident plane wave, the Poynting vector of the scattered waves, directed towards to the core of the plasma, is spatially inhomogeneous. These effects cannot be described within the geometric optics approximation.

From experimental observations, the speed of propagation of the filaments and blobs is of the order of $5 \times 10^3$ m s$^{-1}$ [2, 3]. This is orders of magnitude slower than the group velocity, typically near the speed of light, of the usual RF waves used in experiments. The frequency of temporal variation of the fluctuations is less than 1 MHz, which is at least an order of magnitude slower than the frequency of the RF waves. Consequently, in the model, the filament is treated as stationary, with the density inside the filament being uniform in time.

The approach taken to develop the scattering model in this paper can be broadly stated as follows. The formalism, guided by the filament, is carried out in the cylindrical coordinate system. The Faraday-Ampere equation for the electromagnetic fields inside and outside the filament uses the anisotropic, cold plasma permittivity. Its form is that of the vector Helmholtz equation which can be solved by classical techniques using the vector cylinder functions [11]. The electromagnetic fields of the incident plane wave, the scattered waves, and the waves inside the filament are all expressed in terms of the vector cylinder functions. The boundary conditions at the interface between the filament and the background plasma couple the amplitudes of these waves. For each azimuthal mode number, a set of four, coupled, algebraic equations are obtained. The polarizations of the waves and their propagation characteristics are all included in these coupled equations. The four-equation set can be solved for the amplitudes of the fields inside and outside the filament in terms of the amplitude of the incident wave. The total fields are obtained by summing up the contributions from all the azimuthal components.

2 The analytical full-wave model of scattering

What follows is an outline of the essential features of the theory that underlies the scattering model.

A cylindrical filament, having a circular cross-section, is assumed to be infinitely long in the axial $z$-direction. The approximation for is valid provided the width of the RF
beam along the axial direction is smaller than the length of the filament. The ambient magnetic field, in which the background plasma and the filament are immersed, is taken to be uniform and also along the \( z \)-direction (FIG. 1). The plasma inside and outside the filament is assumed to be a cold fluid, and is described by the continuity and momentum equations for the electrons and ions [12]. The electromagnetic fields of waves in the plasma are given by the Faraday’s and Ampere’s equations in Maxwell’s equations. The system of equations is linearized by assuming that the unperturbed plasma state, in the absence of RF waves, is homogeneous in space and stationary in time, both inside and outside the filament. However, the densities in the two regions are allowed to be arbitrarily different leading to a discontinuous change in density across the boundary separating the filament from the background plasma. The perturbed state is assumed to have a plasma density and RF electromagnetic fields that are varying in space and time. For a time variation of the form \( e^{-i \omega t} \), where \( \omega \) is the angular frequency and \( t \) is the time, the propagation of RF waves in a cold plasma is obtained by combining the Faraday’s and Ampere’s equations,

\[
\nabla \times (\nabla \times \mathbf{E}(\mathbf{r})) - \frac{\omega^2}{c^2} \mathbf{K}(\mathbf{r}) \cdot \mathbf{E}(\mathbf{r}) = 0, \tag{1}
\]

where \( c \) is the speed of light, and \( \mathbf{K}(\mathbf{r}) \) is the plasma permittivity tensor. In the cylindrical coordinate system where the ambient magnetic field \( \mathbf{B}_0 = B_0 \hat{z} \) is aligned along the \( z \)-axis, \( \mathbf{K}(\mathbf{r}) \) has the form [12]

\[
\mathbf{K} = \begin{pmatrix}
K_\rho & -iK_\phi & 0 \\
iK_\phi & K_\rho & 0 \\
0 & 0 & K_z
\end{pmatrix}, \tag{2}
\]
where
\[
K_\rho = 1 - \frac{\omega_{pe}^2}{\omega_0^2 - \omega_{ce}^2} - \sum_i \frac{\omega_{pi}^2}{\omega_0^2 - \omega_{ci}^2},
\]
\[
K_\phi = -\frac{\omega_{ce}}{\omega} \frac{\omega_{pe}^2}{\omega_0^2 - \omega_{ce}^2} + \sum_i \frac{\omega_{ci}}{\omega} \frac{\omega_{pi}^2}{\omega_0^2 - \omega_{ci}^2},
\]
\[
K_z = 1 - \frac{\omega_{pe}^2}{\omega_0^2} - \sum_i \frac{\omega_{pi}^2}{\omega_0^2},
\]
(3)

\(\omega_{pe}\) (\(\omega_{pi}\)) and \(\omega_{ce}\) (\(\omega_{ci}\)) are the angular electron (ion) plasma frequency and cyclotron frequency, respectively, and the index \(i\) represents all the ion species in the plasma. The plasma and cyclotron frequencies can, in general, be functions of space. The permittivity tensor of the background plasma and of the filament are expressed in terms of their respective ion compositions and constant, but different, densities. Subsequently, the elements of \(K\) are constants in each region.

The electromagnetic wave equation (1) has the form of a vector Helmholtz equation. It is solved in the cylindrical coordinate system using the vector cylinder functions [8]. Since the density is constant for the background plasma and for the plasma inside the filament, and since the magnetic field is uniform throughout, (1) along with (2) are solved in each domain separately. The self-consistent solution for the scattered fields and the fields inside the filament is obtained by matching boundary conditions at the interface separating the filament plasma from the background plasma. Assuming that there are no free charges and currents in the interface between the filament and the background plasma, Maxwell's equations impose the following conditions [9],

\[
\hat{\rho} \cdot (\mathbf{D}_I + \mathbf{D}_S) \bigg|_{\rho=a} = \hat{\rho} \cdot \mathbf{D}_F \bigg|_{\rho=a},
\]
(4)

\[
\hat{\rho} \cdot (\mathbf{B}_I + \mathbf{B}_S) \bigg|_{\rho=a} = \hat{\rho} \cdot \mathbf{B}_F \bigg|_{\rho=a},
\]
(5)

\[
\hat{\rho} \times (\mathbf{E}_I + \mathbf{E}_S) \bigg|_{\rho=a} = \hat{\rho} \times \mathbf{E}_F \bigg|_{\rho=a},
\]
(6)

\[
\hat{\rho} \times (\mathbf{B}_I + \mathbf{B}_S) \bigg|_{\rho=a} = \hat{\rho} \times \mathbf{B}_F \bigg|_{\rho=a}.
\]
(7)

where \(\hat{\rho}\) is the unit vector along the radial direction in the cylindrical coordinate system. The subscripts \(I\), \(S\), and \(F\) refer to the incident, scattered, and filamentary wave fields, respectively, \(\mathbf{D} = \epsilon_0 \mathbf{K} \mathbf{E}\) is the wave electric displacement field, \(\epsilon_0\) is the free-space permeability, and \(\mathbf{E}\) and \(\mathbf{B}\) are the wave electric and magnetic field, respectively. The four sets of boundary conditions follow from Gauss' law, Gauss' magnetism law, Faraday's law, and Ampere's law, respectively. The left and right sides of Eqs. (4) – (7) are evaluated at the boundary of the filament \(\rho = a\). Of the six boundary conditions that comprise (4) – (7), it can be shown that, in a cold plasma, only four are independent [8].

The analytic treatment of the scattering is carried out as follows. A plane wave, propagating in a background plasma with a prescribed frequency and \(k_z\) – the component
of the wave vector along the magnetic field line – is incident on the cylindrical filament. Since the axis of the filament is along the magnetic field line, $k_z$ remains the same for the fields inside the filament and the scattered fields [8]. The scattered wave fields are restricted to be outgoing waves; i.e., their radial group velocity is in a direction away from the filament. In case the scattered fields are not propagating waves, they are restricted to spatially decaying fields. Inside the filament, no such restrictions are imposed. With these assumptions, the electromagnetic fields over all space are uniquely determined for a given amplitude of the electric field of the incident plane wave.

Some of the features that result from the scattering model are best displayed by the time-averaged Poynting vector [11],

$$\langle S(t) \rangle = \frac{1}{2} \text{Re}(E \times H^*),$$

where $\langle \ldots \rangle$ denotes the time average, and $H^*$ is the complex conjugate of the magnetic intensity $H$. In a plasma, the wave magnetic field $B$ is related to $H$ by $B = \mu_0 H$, where $\mu_0$ is the free-space permeability. Since all the electromagnetic fields are proportional to the amplitude of the electric field of the incident plane wave, it is more appropriate to quantify the normalized Poynting vector

$$P = \frac{\langle S(t) \rangle}{|\langle S_i \rangle|},$$

where $|\langle S_i \rangle|$ is the magnitude of the time-averaged Poynting vector of the incident plane wave.

The theoretical model and analysis are valid for waves in any frequency range, including the electron cyclotron, lower hybrid, and ion cyclotron waves.

### 3 Results and discussion

The results that follow are for a plasma composed of electrons and deuterons in a magnetic field $B_0 = 4.5$ T. The incident RF plane wave is in the electron cyclotron range of frequencies and has a frequency of 170 GHz. It is assumed to be propagating normal to the direction of the magnetic field along the positive $x$-direction. The $x$-direction is directed towards the core of the toroidal plasma and the $y$-component is along the poloidal direction. The electron density of the background plasma and of the plasma inside the filament is taken to be $n_b = 2 \times 10^{18}$ m$^{-3}$ and $n_f = 3.2 \times 10^{18}$ m$^{-3}$, respectively, which implies that $\delta n = 0.6$. We assume that the incident plane wave is the extraordinary X mode [12]. The Cartesian components of the Poynting vector for the incoming wave are, $P_i^x = 1$, $P_i^y = 0$, and $P_i^z = 0$.

FIG. 2 shows the contour plot for $P_x$ (left pane) and $P_y$ (right pane) in the $x - y$ plane. The filament has a radius $a = 1$ cm, centered at $x = y = 0$, with its cross-section displayed as a white circle. The X wave is incident from the left-hand side ($x < 0$). Since $P_x$ is positive, the power flow is in the positive $x$-direction. The uniformity in the spatial region $x < 0$ indicates that there is no backscattering of the incident wave due to the
FIG. 2: Contours of $P_x$ (left) and $P_y$ (right) in the $x - y$ plane for an incident X wave. The circular cross-section of the filament has a radius of 1 cm, and is indicated by the white circle. The relevant parameters are given in the text.

filament. The scattering in the forward direction, and the power flowing into the plasma, is not spatially uniform in the $y$-direction, which would have been the case if the filament did not exist. The spatial redistribution of power along the $y$-axis is within about $\pm 10\%$ of the incident power as indicated by the scale on the right-hand side of the figure. The non-uniformity in $P_x$ is due to diffraction, and constructive and destructive interference between the incident and the scattered waves. This leads to shadowing in regions where $P_x$ is less than 1, and focusing in regions where $P_x$ is greater than 1. The shadowing and focusing depends on the density of the plasma inside the filament. For higher densities, these effects are more pronounced.

The right pane in FIG. 2 shows the component of the Poynting flux in the $y$-direction. From this figure it is quite apparent that some of the incident power is side-scattered by the filament. The side-scattering couples power to surface waves. The scattering also leads to a change in the polarization of the scattered wave, so that some of the power gets coupled to the ordinary O wave.

If, instead, the incident wave is the O wave, the results (not shown) are similar to those shown in FIG. 2. Again, the scattering is in the forward direction with essentially no reflected power. The power is spatially fragmented just as it is for an incident X wave.

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