Kinetic Electron and Ion Instability of the Lunar Wake Simulated at Physical Mass Ratio

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Kinetic Electron and Ion Instability of the Lunar Wake Simulated at Physical Mass Ratio

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The solar wind wake behind the moon is studied with 1D electrostatic particle-in-cell (PIC) simulations using a physical ion to electron mass ratio (unlike prior investigations); the simulations also apply more generally to supersonic flow of dense magnetized plasma past non-magnetic objects. A hybrid electrostatic Boltzmann electron treatment is first used to investigate the ion stability in the absence of kinetic electron effects, showing that the ions are two-stream unstable for downstream wake distances (in lunar radii) greater than about three times the solar wind Mach number. Simulations with PIC electrons are then used to show that kinetic electron effects can lead to disruption of the ion streams at least three times closer to the moon than in the hybrid simulations. This disruption occurs as the result of a novel wake phenomenon: the non-linear growth of electron holes spawned from a narrow dimple in the electron velocity distribution. Most of the holes arising from the dimple are small and quickly leave the wake, approximately following the unperturbed electron phase-space trajectories, but some holes originating near the center of the wake remain and grow large enough to trigger disruption of the ion streams. Non-linear kinetic-electron effects are therefore essential to a comprehensive understanding of the 1D electrostatic stability of such wakes.

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I. INTRODUCTION

The supersonic flow of the solar wind past the moon and other objects without global magnetospheres is a topic of continued theoretical and experimental interest. The ongoing ARTEMIS mission is studying the solar wind in the vicinity of the moon, with targeted 3D hybrid simulations being used to interpret the observed fluctuations. More generally, simulations of dense magnetized plasma flow past non-magnetic objects has wide applicability, ranging from probes in laboratory plasmas to asteroids and the moon.

Solar wind flow past the moon leads to an elaborate lunar plasma environment and wake. At distances beyond a few lunar radii the main dynamics of interest are those associated with the ambipolar plasma expansion into the void trailing the moon, and in particular any resulting instabilities. This process has been studied with fully kinetic simulations in 1D and 2D, as well as with hybrid simulations in 3D. There are also 2D kinetic simulations which ignore the magnetic field of the solar wind, but these are less relevant to the moon given its large size relative to the gyroradius scales in the solar wind.

Despite the wide range of prior simulations, none have so far accurately captured the 1D electrostatic phenomena which can occur in the wakes of large non-magnetic objects. Birch and Chapman attributed the fluctuations they (and previously Farrell et al.) observed to electron beam instability. However, Hutchinson showed that the unphysical ion to electron mass ratio used in those simulations artificially enhances the energy scale of electron–electron instability, indicating that a realistic mass ratio is needed to accurately model such instabilities in the wake. The present work confirms the importance of using a realistic mass ratio, discovering a novel wake phenomenon where electron holes grow from the small energy scale of the electron–electron instabilities to a scale where they can disrupt the counterstreaming ion beams in the wake. This observation has already inspired theoretical investigations into the underlying mechanism, which show that the changing density in the wake can act as a drive for hole growth.

Kinetic electron effects are thus shown to be important in the flow of magnetized plasmas past objects much larger than the plasma Debye length. The case of supersonic flow is treated here, but the failure of Boltzmann or other hybrid electron treatments to capture the relevant electrostatic phenomena may well extend to cases with slower plasma flow: counterstreaming ions will also be present for slower flow, and the electrons will still see a varying potential and local density, so the main differences are geometrical and related to boundary conditions at the collecting object. Whether these differences are enough to render the kinetic electron effects unimportant is a topic that should be investigated, as it may be relevant to Mach probes and other means of invasive plasma measurements.

This paper is organized as follows: The 1D electrostatic simulation approach used is described in Section II. Simulations using Boltzmann electrons are presented in Section III, showing the onset of ion two-stream insta-
bility; the unstable (but mass-ratio dependent) electron distributions associated with the Boltzmann-simulated wake are also illustrated. Fully kinetic simulations are presented in Section IV, where some of the electron holes formed early in the wake are seen to grow and lead to disruption of the ion streams much closer to the moon than in the Boltzmann-electron simulations. Concluding remarks are given in Section V.

II. 1D ELECTROSTATIC SIMULATION METHOD

The lunar wake simulation is reduced to 1D through the same approach taken in prior such studies\textsuperscript{5–7}; by following the plasma in the frame moving with the solar wind, where the perpendicular motion is restricted by the magnetic field. What is simulated is then the parallel motion of electrons and ions along an unperturbed magnetic field line as a function of time. For a static wake this time-dependence can be translated to a spatial dependence by following the trajectory of the field line, and a single 1D simulation thus applies to a wide range of solar wind flow velocities and magnetic field orientations with respect to the moon\textsuperscript{16} (in suitably scaled coordinates).

For ease of comparison with the prior studies\textsuperscript{5–7} the relative motion of the solar wind and the moon is taken to be perpendicular to the magnetic field and have speed $v_{\text{sw}} = 25 c_s$. Here $c_s = \sqrt{T_e/m_i}$ is the (cold ion) sound speed, which is equal to the ion thermal velocity since $T_e = T_i$. The distance behind the moon is thus $x = v_{\text{sw}} t$, and the position parallel to the magnetic field is denoted $y$. The particular choice of solar wind speed does not enter into the simulations, only the representation of the results in the figures, so other speeds can be accommodated by a simple scaling of $x$. Note that the convention for $x$ used herein differs from that used in References 16 and 17 by a factor of the (cold ion) Mach number $v_{\text{sw}}/c_s = 25$.

A 1D electrostatic particle-in-cell (PIC) code, referred to here as ESPIC, has been developed to carry out the simulations. The ions and electrons are evolved with a standard leap-frog scheme at time steps small enough to resolve the electron motion, and their charge distributed to the two adjacent nodes of the mesh using the cloud-in-cell approach\textsuperscript{18}. At each time-step the finite-difference Poisson equation is solved for the potential using a direct tridiagonal method, yielding a constant electric field between any two mesh nodes to accelerate the particles. The potential at the boundaries is set to zero with homogeneous Dirichlet boundary conditions, and particles are injected there according to the assumed Maxwellian background distributions.

ESPIC can be run in parallel to accommodate the large number of particles and time-steps needed to resolve the wide ranges of relevant length and time scales. Even so, large scale-separations such as in the case of the lunar wake present a challenge: the lunar radius $R_M$ is 1730 km, while the ion thermal gyroradius is around 40 km and the electron and ion Debye lengths are around $7 \text{ m}^{19,20}$. To resolve the Debye length in a domain many lunar radii across would thus require $\sim 10^7$ grid-points, allowing for a few grid-points per Debye length to reduce discretization error. That is not currently feasible using a realistic ion to electron mass ratio, but extrapolation based on simulations at larger (but still small) Debye lengths can yield insight.

For such 1D electrostatic simulations to be applicable, the main influence of the magnetic field on the particles must be to restrict their motion in the directions perpendicular to it. Further, the collecting object (e.g., the moon) must have small apparent extent in the direction of flow, such that the ions do not move much as the field line traverses the object. This can either be because the flow is sufficiently supersonic or because the object has a large aspect ratio, and in either case the detailed shape of the object doesn’t matter; its main influence is to leave a region depleted of plasma behind it. The parallel dynamics of the ambipolar plasma expansion into that depleted region then dominate the solution, and can be captured in a 1D domain by suddenly removing the ions and electrons from the part of that domain crossed by the object; this is the approach taken in ESPIC.

Despite not directly influencing the ions, the shape (and surface charge) of the object could still affect the (much faster) electrons. For example, in the case of the lunar wake the electron thermal speed is faster than the solar wind flow (for a realistic mass ratio), so it is not very accurate to model the passage of the moon by a sudden removal of electrons. However, the simultaneous removal of both species provides a convenient way to remove an equal number of electrons and ions, which avoids unphysical charge build-up associated with inaccurate object boundary conditions (especially important for simulations with short Debye length). Further, as will be shown in Section III, the initial electron disturbance (and thus any inaccuracy associated with it) quickly propagates out of the domain in simulations with a realistic mass ratio. Thus, the way of implementing the (2D) passage of the moon in the 1D ESPIC simulations is thought to accurately capture the relevant effects on the part of the wake of interest in the present work.

The main differences between the ESPIC simulations and prior 1D kinetic simulations\textsuperscript{5–7} are finer phase-space resolution (up to $2 \times 10^8$ grid points and $10^9$ particles), the use of a physical mass ratio, and the absence of periodic boundary conditions. Preventing the wake from interacting with itself through the periodic boundaries set a lower limit on the domain size in the prior simulations, and for a physical mass ratio that constraint would be unmanageable. Care must still be taken to avoid reflection of potential structures from the boundaries, but that is much less restrictive in terms of domain size than the periodic boundaries; if need be absorbing potential boundary conditions could be added allow even smaller domains.

For small Debye length $\lambda_D$, a random initialization of
FIG. 1. Ion density $n_i$ (in units of solar wind density $n_\infty$), normalized charge density $n_i - n_e$, and potential $\phi$ of the wake simulated with Boltzmann electrons for $\lambda_{De} = 0.02R_M$ (with the illustrative plasma flow speed $v_{sw} = 25c_s$).

particles in the PIC method yields unphysically large fluctuations in the initial potential. This is especially true at long wave-lengths, as the amplitude of potential fluctuations of wavelength $\lambda$ can be shown to scale with $(\lambda/\lambda_{De})^{3/2}$ for Gaussian counting statistics. For the same number of particles, the amplitude of the potential fluctuations can be dramatically reduced by using a quiet start18, where particle positions and velocities are initialized more uniformly than in a true random start; a quiet start is used in ESPIC.

III. BOLTZMANN ELECTRON SIMULATIONS

Hybrid simulation approaches involve the use fluid or other non-kinetic treatments of electrons to greatly reduce the computational cost of simulations, which then only need to resolve the ion time-scales as opposed to the (typically much shorter) electron ones. One such approach is to assume that the electron density along a magnetic field line satisfies a Boltzmann relation

$$n_e = n_{e0} \exp \left( \frac{e\phi}{T_e} \right),$$

which (for instance) it would exactly along a static, monotonically decreasing (repulsive) potential from some point with a stationary Maxwellian electron distribution of density $n_{e0}$. Such Boltzmann electrons are implemented in ESPIC by solving the non-linear Poisson equation arising from using Equation 1 for the electron density, and are used in the simulations throughout this section (even when moving electrons kinetically to examine their distribution).

The wake structure resulting from a simulation with Boltzmann electrons and a solar wind electron Debye length $\lambda_{De} = 0.02R_M$ is shown in Figure 1. As discussed in Section II a solar wind speed $v_{sw} = 25c_s$ is used to facilitate comparison with prior 1D kinetic simulations5–7. The Boltzmann-electron simulation captures the main features of the initial plasma expansion into the void behind the moon seen in the prior simulations: A negative potential in the density-depleted region approximately balances the electron and ion densities, with significant charge density seen only in the transition from unperturbed plasma to the strongly density-depleted region. Later in the wake, for $x \gtrsim 140R_M$, the simulation shows rapid ion (and charge) density variations (verti-

FIG. 2. Ion distribution at three different times (and thus $x$) in the simulation shown in Figure 1.
FIG. 3. Ion density in the wake for Boltzmann-electron simulations at three different \( \lambda_{D_e} \).

FIG. 4. Ion distribution at \( x = 82.5 R_M \) and \( y = 5 R_M \) in Figure 2, with Gaussian counting errors for each bin shown in red.

cally elongated striations in the plots) resulting from ion two-stream instability (sometimes referred to as the ion–ion [acoustic] instability \(^{21,22}\)).

Figure 2 shows the ion phase space for the Boltzmann-electron simulation at three different times/positions. The main initial response of the ions is that they are accelerated towards the void by the negative potential structure. Later in the wake free-streaming of ions is the dominant process, distorting the initial phase space void such that the density-depletion is shallower but more spread out. This sets up a configuration of two ion beams across most of the wake, and the velocity gap between the two beams gradually shrinks. Eventually the the ion two-stream instability grows to a non-linear stage, where large ion holes form and give rise to the ion density variations seen in Figure 1.

Simulations with shorter Debye length show instability-related density variations earlier in the wake, as can be seen in Figure 3. The large-scale structure of the wake is not sensitive to the change in Debye length, so the earlier appearance of the non-linear features of the two-stream instability is caused mainly by the faster growth associated with the higher plasma frequency at shorter Debye length; it takes a significant time for unstable perturbations to grow large enough to be visible when the Debye length is not infinitesimal. Extrapolating to very small Debye-lengths indicates that the ion beams become two-stream unstable when \( t \approx 3 R_M / c_s \), which corresponds to a distance of \( \sim 75 R_M \) behind the moon for the particular solar wind speed used in the figures.

Traditionally, Maxwellian beams are considered when studying the two-stream instability (e.g., Ref. 21), and cold ion beams are found to be unstable when their separation is less than about twice the sound speed. However, as illustrated in Figure 4 the ion beams in the simulated wake are not Maxwellian, and have sharp edges associated with the initial void. To include enough particles to resolve the fine features in velocity-space, large spatial bins of width \( 4 \lambda_{D_e} \) are used when storing the distributions for the following stability analysis, leading to a small amount of smoothing.

The electrostatic ion stability at each location in the wake has been analyzed by calculating numerically the complex Fourier-transformed susceptibility of the (slightly further smoothed) ion distribution at that location together with Maxwellian electrons, in a 1D linearized Vlasov–Poisson system, and setting it equal to zero. The ion–electron instability is suppressed in this calculation by artificially setting the imaginary part of the electron susceptibility to zero; this is appropriate because a Boltzmann electron treatment excludes electron Landau damping or growth. Important deviations of the electron distribution from a Maxwellian will be
unphysical) sudden removal of electrons from the computational domain. The disturbance propagates out of the domain very quickly at realistic mass ratios, so any inaccuracy associated with the 1D treatment of the passage of the moon is unlikely to affect the parts of the wake of interest in the present work.

IV. FULLY KINETIC SIMULATIONS

Protons are 1836 times more massive than electrons, providing a lower bound for the true ion to electron mass ratio $m_i/m_e$ in a typical plasma. Simulations run at such mass ratios are computationally expensive, so a common approach is to use an unphysically small mass ratio in an attempt to capture the essential physics of the scale separation without paying the full computational cost. This was done in prior PIC simulations of the lunar wake\cite{5–8}, which used the highly unphysical mass ratio $m_i/m_e = 20$. Later, Hutchinson\cite{16} showed that an unphysical mass ratio greatly enhances the degree to which the lunar wake electron distribution is unstable, and Hong et al.\cite{24} showed that an unphysical mass ratio can significantly affect ion-driven beam instabilities in ways that are not easily scaled to a physical mass ratio. Acknowledging these findings, the present work uses simulations at unphysical mass ratios only to illustrate phenomena observed in simulations using the physical mass ratio for hydrogen.

At the true mass ratio and $\lambda_{De} = 0.0025 R_M$, treating both electrons and ions kinetically in ESPIC gives rise to the ion density shown in Figure 7. Treating the electrons kinetically gives rise to instability much earlier than was seen in the simulations with Boltzmann electrons (cf. Figure 3). Non-linear disruption of the ion beams in this case occurs at $t \approx 1.1 R_M/c_s$, which corresponds to a distance of $\sim 28 R_M$ behind the moon (at the particular solar wind speed used for illustration). The nature of the density variations is also different, with fewer, less periodic
density enhancements gradually widening.

For the same simulation, the evolution of the electron distribution is shown in Figure 8. Very early in the wake, the only noticeable features are the initial void and the (faint) outward propagating initial disturbance from the sudden removal of electrons. Later in the wake a narrow dimple is seen, showing that the dimple persists in some form also when self-consistently considering electron instabilities (cf. Figure 6). Eventually, electron holes with large velocity extent form and gradually widen in spatial extent, corresponding to the density enhancements seen in Figure 7.

Because simulations at the true mass ratio have an extremely narrow dimple and are somewhat noisy, a simulation using $m_i/m_e = 459$ (but the same Debye length) is now used to illustrate the relevant electron phenomena. To better visualize the dimple and holes, a Maxwellian of the same density (and the solar wind electron temperature) is subtracted from the electron distribution at each point in space. The result is shown in Figure 9, for a time before electron holes with large velocity-extent develop. Small hole-like perturbations are seen along the dimple in the electron distribution, and examining consecutive time-steps reveals that they convect outwards along the dimple in phase space (without significantly perturbing the ions).

At a slightly later time, an almost stationary electron hole near $y = 0$ starts growing in velocity extent; the hole moves slowly ($v_y \sim -c_s$) towards negative $y$, not appearing to accelerate to follow the dimple like the other holes. The ion and electron distributions in the central region of the domain are shown in Figure 10, for a time when the hole has grown to a large enough extent to significantly perturb the ion beams. After that time the ion beams are quickly disrupted (similarly to in prior simulations\textsuperscript{5,7}), further widening the electron hole in velocity as the gap between the ion beams closes. A mechanism has been proposed to explain the growth of such holes\textsuperscript{17}, whereby the increasing local density as the hole moves downstream drives an expansion of its velocity-extent.

Returning to the simulation at the true mass ratio,
a careful examination of the evolution of the electron distribution reveals that the large electron holes seen in Figure 8 arise in the same way as the one discussed for the slightly reduced mass ratio: small holes emerge from the dimple near $y = 0$, move slowly away from the dimple while growing in velocity-extent and remaining centered at small $v_y$, and then eventually expand to their full velocity-extent as they disrupt the ion beams; the holes then keep expanding in spatial extent as seen in Figure 7. The first seed of a large hole emerges at a time corresponding to $x \approx 14R_M$, where the local electron density is $0.21n_\infty$, and the holes grow until they disrupt the ions around $x = 28R_M$, at which point the local density is in the range $0.46n_\infty$ to $0.56n_\infty$ (depending on the hole position). For the same time-interval the total velocity-separation of the ion beams shrinks from $\sim 5c_s$ to $\sim 3c_s$. That density increase and beam-separation decrease is sufficient for the proposed hole growth mechanism $^{17}$ to lead to ion-beam disruption, consistent with the simulations.

Because the electron holes do not remain exactly at $y = 0$ the ion beams are not quite balanced, as illustrated in the (extreme) example in Figure 4. Further, the finite velocities of the electron holes mean that they are likely closer to one of the beams in velocity than to the other, and those velocity-separations evolve as time passes. The situation in the simulations is thus not quite that of the symmetric cold beams considered in the proposed hole growth mechanism $^{17}$, but the theory nonetheless seems to capture the main features of the hole growth and beam disruption. Where in the wake the ion beams are disrupted thus depends on two main factors: the amount of electron hole growth from the increasing local density, and the decreasing hole size needed to disrupt the ion beams as their velocity-separation shrinks.

Further simulations have been carried out to explore the sensitivity of the ion beam disruption to changes in mass ratio and electron Debye length. In general, decreasing the Debye length or the ion to electron mass ratio leads to earlier disruption of the ion beams. For example, changing the mass ratio to $m_i/m_e = 115$ leads to the ion beams being disrupted at a time corresponding to $x \approx 12R_M$ when they are separated by $\sim 5.5c_s$. It is presently not computationally feasible to do true mass ratio simulations with ESPIC at Debye lengths much smaller than the $\lambda_{De} = 0.0025R_M$ of the simulations presented in this section, so the $\lambda_{De} \sim 10^{-5}R_M$ of the solar wind remains out of reach. However, the observed trend is for there to be more electron holes at shorter Debye lengths, appearing and disrupting the ion beams even earlier in the wake; at $\lambda_{De} = 0.00125R_M$ disruption occurs at $x \approx 20R_M$.

V. CONCLUSIONS

The present work highlights the importance of using a realistic ion to electron mass ratio in 1D simulations of magnetized supersonic wakes behind non-magnetic objects. Doing so has revealed a novel phenomenon where the velocity extents of electron holes grow from the small scale associated with electron-electron instability to one where the holes can significantly perturb the ions. As a result, the dimple in the electron distribution remains important despite its narrow width at realistic mass ratios, acting as a seed of electron holes which eventually disrupt the counterstreaming ion beams in the wake much earlier than in the Boltzmann-electron simulations.

Prior kinetic simulations$^{7,8}$ observed strong kinetic electron effects at a highly artificial mass ratio ($m_i/m_e = 20$), but could not establish the importance of those effects at physical mass ratios because of the strong mass-ratio dependence of the dimple$^{16}$. The present results demonstrate unequivocally the importance of kinetic electron effects in electrostatic 1D simulations, thus calling into question the validity of the hybrid simulations frequently used in attempts to explain experimental observations. It is conceivable that other physical effects suppress the kinetic electron effects: for example, magnetic perturbations have been shown to be more important for solar wind flow which is far from perpendicular to the magnetic field$^{10}$, and so could in principle interfere with the parallel electron and ion dynamics. Unless such a suppressing mechanism can be identified, however, accounting for kinetic electron effects on ion stability appears to be essential.

Because of the small size of the ion thermal gyroradius compared with the radius of the moon, the 1D treatment is likely resonably accurate for the early large-scale structure of the wake. However, the Debye length is significantly smaller than even the electron thermal gyroradius, so the accuracy of the 1D non-linear instability evolution is less certain. In principle the electrostatic modes present in 1D could couple to electromagnetic modes, possibly at oblique angles, giving the instabilities a multi-dimensional nature and introducing pitch-angle scattering, which could affect the electron dimple. Further, the growing electron holes may not be stable for the true ordering of length scales, possibly interfering with the ability of electron holes to reach the size needed to disrupt the ion beams. Kinetic simulations of higher dimension are thus needed, though these will be computationally challenging given the need to use a realistic mass ratio and a short Debye length.

Though the lunar wake has been used as the main example of possible applicability of the present simulations, they solve the general problem of supersonic flow past a non-magnetic object. As such they apply equally well to probes in magnetized plasmas and other large bodies in the solar wind, provided the main assumptions hold. The importance of electron kinetic effects at supersonic flow also raises the question of the extent to which similar phenomena can occur at slower magnetized plasma flow, but to address that question will require multi-dimensional simulations since the finite extent of the object in the flow direction and its boundary conditions then matter.
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