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Robotic Calibration of the Motional Stark Effect Diagnostic on Alcator C-Mod

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The capability to calibrate diagnostics, such as the Motional Stark Effect (MSE) diagnostic, without using plasma or beam-into-gas discharges will become increasingly important on next step fusion facilities due to machine availability and operational constraints. A robotic calibration system consisting of a motorized three axis positioning system and a polarization light source capable of generating arbitrary polarization states with a linear polarization angle accuracy of < 0.05° has been constructed and has been used to calibrate the MSE diagnostic deployed on Alcator C-Mod. The polarization response of the complex diagnostic is shown to be fully captured using a Fourier expansion of the detector signals in terms of even harmonics of the input polarization angle. The system’s high precision robotic control of position and orientation allow it to be used also to calibrate the geometry of the instrument’s view. Combined with careful measurements of the narrow bandpass spectral filters, this system fully calibrates the diagnostic without any plasma discharges. The system’s high repeatability, flexibility and speed has been exploited to quantify several systematics in the MSE diagnostic response, providing a more complete understanding of the diagnostic performance.

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The Motional Stark Effect (MSE) line polarization diagnostic\textsuperscript{1} measures the local electric field direction by collecting polarized light emitted from a hydrogenic neutral beam injected into a magnetically confined plasma. Line emission from the neutrals is spectrally split into a multiplet by the motional Stark effect\textsuperscript{2} from the strong Lorentz electric field in the atom’s frame ($\vec{E}_{\text{Lorentz}} = \vec{v}_{\text{beam}} \times \vec{B}$) as the atoms cross the strong magnetic field at high velocity. When viewed in a direction perpendicular to $\vec{E}$ the emission consists of two orthogonally polarized components ($\sigma$, $\pi$) which are polarized perpendicular and parallel to $\vec{E}$ respectively. The emitted light is collected and spectrally filtered for one multiplet component whose polarization angle is measured, thus yielding information about the magnetic and electric fields at the point of emission. The Balmer-$\alpha$ transition is typically used since it is has significant intensity in the visible range. Many polarization-based MSE systems employ a dual photo-elastic modulator (PEM) based polarimeter\textsuperscript{3} which encodes the polarization of the light into a time-varying signal by passing the light through a pair of vibrationally stressed fused silica plates followed by a linear polarizer. The stress-induced birefringence in the plates modifies the polarization before it is passed through the polarizer producing intensity modulations at the harmonics of the PEM’s. The amplitude of the signal detected at harmonics of the PEM’s vibration frequencies is then demodulated using a lock-in amplifier or a numerical phased-locked loop\textsuperscript{4}, thereby yielding the Stokes parameters of the radiation incident on the polarimeter. The diagnostic technique has been employed on numerous tokamaks\textsuperscript{5–11} to determine the poloidal magnetic field and/or the radial electric field at the intersection of the sight-line and the neutral beam. The measured magnetic field direction is then utilized to constrain magnetic reconstructions of the plasma equilibrium.

The MSE diagnostic on the Alcator C-Mod tokamak\textsuperscript{12} consists of a complex in-vacuum periscope; (five lenses and three dielectric mirrors) that conveys the polarized emission through a vacuum window and then through an additional four lenses before it is incident on the dual-PEM based polarimeter. Ten sets of 16 optical fibers then collect the time-varying emission at the image plane of the periscope and convey it to a diagnostic hall $\sim 20$ meters away. Each of these ten sightlines is spectrally filtered using narrow bandpass filters (FWHM 0.5 – 1.0 nm) and the emission is detected by avalanche photodiodes (APDs). The signals are digitized at 1 MHz and later demodulated using a numerical phase-locked loop. The optical components of the periscope are optimized to preserve the polarization until it can be encoded by the PEMs, however the components do modify the polarization somewhat.
before it enters the polarimeter. This diagnostic response, the action of the PEM based polarimeter, the geometry of the view and the bandpass of the spectral filters all require careful calibration.

The remainder of this paper summarizes previously demonstrated calibration techniques and then describes a robotic calibration system and its use for the calibration of the MSE diagnostic on Alcator C-Mod. The repeatability of the technique and limitations are discussed as well as its potential use to study systematic errors and sensitivities in the diagnostic.

I. MOTIONAL STARK EFFECT DIAGNOSTIC CALIBRATION

A complete calibration of a PEM based MSE diagnostic involves three major components:

• Determine the geometry factors that relate the polarization angle incident on the diagnostic to the components of the plasma’s magnetic and electric fields;\textsuperscript{13}

• Quantify the relationship between the measured intensities at the PEM harmonics and the polarization of the light incident upon the diagnostic objective lens, particularly the linear polarization angle; and

• Measure the spectral transmission of the narrow bandpass filters used to isolate either the $\sigma$ or $\pi$ component of the beam emission and qualify the technique used to control the bandpasses.

In a conventional aspect ratio tokamak, the MSE diagnostic must be calibrated to yield measured polarization angles relative to the tokamak’s toroidal field direction with an accuracy of 0.1° to accurately constrain magnetic reconstructions. The orientation of the tokamak vacuum magnetic field is typically well known relative to gravity and to the other magnetic diagnostics\textsuperscript{14,15}, thus the MSE diagnostic is calibrated with respect to the local gravity direction.

In many tokamaks, the MSE calibration is determined using a beam-into-gas technique\textsuperscript{1} in which the neutral beam is fired into a gas-filled torus with the tokamak magnet system energized. The calibration that relates measured PEM signals to the applied vacuum magnetic fields is developed using successive shots or magnetic field ramps. The main advantage of this technique is that the diagnostic is calibrated in a situation very similar...
to the operational conditions during plasma experiments, although the beam spectra has been shown to differ significantly\(^{16}\). Unfortunately, the prompt polarized emission from the beam is partially polluted by emission from secondary beam neutrals. These are neutrals that ionize, then gyrate about the local magnetic field thereby changing their local velocity direction, and then recombine or charge exchange and emit a photon before leaving the viewing volume\(^{17}\). Some of this secondary neutral emission has Doppler shifts and Stark splits such that it falls within the diagnostic’s spectral filters and its polarized components are captured by the diagnostic. This collected emission occurs at random positions during a gyro-motion, thus the Lorentz electric field is not easily related to the background magnetic and electric fields. This emission systematically biases the net collected polarization during a beam-into-gas discharge making the calibration difficult to interpret. The effect is strongest in tokamaks with near radial beam injection such as Alcator C-Mod and increases as the torus gas pressure is increased. Low gas pressures are thus required, thereby increasing the necessary signal integration time. Although progress has been made in understanding the beam-into-gas experimental results, future large tokamaks including ITER may limit operation of beam-into-gas experiments due to potential damage to the plasma-facing components from neutral beam-shine through\(^{18}\).

Other calibration techniques using the neutral beam as a source have been developed in which the edge magnetic pitch angle of the plasma is used to calibrate the MSE sightlines near the edge of the plasma using the known plasma current and data from external magnetic diagnostics. The input polarization angle is scanned at multiple sightlines either through plasma position jogs\(^{19,20}\) or the plasma current is ramped at various plasma sizes\(^{21}\). Although these techniques capture the full diagnostic response they require significant dedicated tokamak plasma operation time to establish a calibration database, thus impacting tokamak availability while only providing a limited calibration range. Additionally, these techniques rely on magnetic reconstructions of the plasma edge which may introduce systematic sources of error.

Both techniques require dedicated machine operation time while providing only a very limited set of calibration data, thus making it difficult to characterize systematic errors and sensitivities in the diagnostic or to explore novel operational modes. It is therefore desirable to develop a calibration technique that can be performed during a maintenance period, removing the impact to tokamak operation due to beam-into-gas or plasma discharges and
allowing the diagnostic response to be studied comprehensively using many high resolution, repeated calibrations. Historically, the discharge-based MSE calibrations on Alcator C-Mod have been complemented by invessel calibrations that illuminate the diagnostic with linearly polarized light from a high precision, rotatable polarized light source. The polarized light source was manually positioned at each of the ten MSE sightlines and manually oriented to illuminate the MSE objective lens. Similar techniques have been implemented on the DIII-D$^{22}$ and JET$^{23}$ tokamaks. This system provides an accurate calibration of the diagnostic response to polarized light (polarization angle accuracy $\sim 0.05^\circ$), but is tedious, requires significant manned time invessel and requires additional techniques to calibrate the diagnostic geometry.

This paper describes the development and use of a robotic calibration system which automatically positions and orients a precision polarization generation source, and other optical heads for specialized measurements, at each MSE sightline and also determines the diagnostic geometry self-consistently. This system decreases the time required for a complete MSE calibration by an order of magnitude relative to the previous manual approach while increasing calibration accuracy. The increased performance facilitates studies of diagnostic effects such as non-ideal performance of the PEM-based polarimeter, variability of the diagnostic response due to birefringence in the relay optics, and the effects of non-ideal mirrors. The technique provides high fidelity calibration of the diagnostic in multiple operational modes to optimize performance. This optimization of the diagnostic operation and evaluation of sources of systematic error in the diagnostic will be important as MSE diagnostics become more complex, are installed in harsher environments and physics studies demand more accurate measurements.

II. ROBOTIC CALIBRATION SYSTEM

This section describes the automated MSE calibration system that was designed and constructed to illuminate the diagnostic with light having a precisely controlled polarization state from a precisely controlled position inside the tokamak and to determine the MSE measurement geometry. The system is comprised of a three-axis positioning system, a high precision polarization generation head and control software.
FIG. 1. Positioning system of the robotic calibration system. A laser is aligned coincident to the beam axis using targets inserted in the beam duct during maintenance periods. The motorized linear translation stage is aligned to this laser and can translate along the low-field side of the beam trajectory. Mounted to the moving element of this stage are two orthogonally orientated motorized rotational stages that move a platform in the pitch and yaw directions with the pivot point centered on the beam axis.

A. Positioning System

Using precision screw adjustments, a stepper-motor actuated linear translation stage with a positioning accuracy of \( \pm 15 \mu m \) and a repeatability of \(< 2.5 \mu m\) is aligned to a laser that traces the neutral beam trajectory. This motorized stage can translate along the entire low-field side beam trajectory inside the torus, covering the entire field of view of the Alcator C-Mod MSE diagnostic. The linear stage carries two stepper-motor actuated rotation stages mounted orthogonally to one another. These rotation stages move a platform in the pitch and yaw directions with a positioning accuracy of \( \pm 0.05^\circ \) and a repeatability of \(< 0.02^\circ \) with the pivot point centered on the beam axis. This system thus presents a platform which can be located at any point along the beam axis on the low-field side of the tokamak and can be oriented to point in any desired direction. The positioning system is shown in Figure 1.
B. Polarization Generation Head

The positioning system typically carries a precision polarization generation head that creates user-defined polarization states. The head consists of a large stepper-motor actuated rotational stage with positioning accuracy of $\pm 0.02^\circ$ with a repeatability $< 0.003^\circ$. An electronic inclinometer is attached to the base of the stage and determines the position of the base relative to gravity to $< 0.02^\circ$ in two orthogonal directions (i.e. pitch and roll). A dichroic linear polarizer is mounted on the calibration side of the rotating element of the stage. A removable, digitally controlled, variable liquid-crystal retarder is optionally mounted in front of the linear polarizer with its fast axis angled $45^\circ$ with respect to the linear polarizer’s transmission axis. The retarder can be varied to impart over a quarter-wave of retardance, thus generating fully circularly polarized light. Two light sources are mounted on the other side of the rotational element. One source consists of a uniformly spaced array of 196 red LEDs (40 nm FWHM centered at 640 nm) behind an optical diffuser. This creates a nearly Lambertian red light source of controllable brightness with a clear aperture of approximately 75 mm diameter. The other source is a red laser diode (5 mW @ 655 nm) mounted behind the LED array and aligned to shine along the rotation axis of the rotational stage through a small hole in the LED array and diffuser. The rotational stage carries the polarizer, retarder, LED array and laser as a single unit as it rotates. The transmission axis of the linear polarizer is calibrated using a Malus law calibration technique detailed in Appendix A. The polarization generation head can thus generate arbitrary Stokes vectors with either a Lambertian or a laser light source, with polarization ellipticity ranging from 0 to 1 with total polarization fraction $> 0.99$ and with an azimuthal polarization angle known to better than $0.05^\circ$ relative to gravity. For standard MSE calibrations the system is typically operated without the variable retarder since the calibration is concerned predominately with linearly polarized light. The polarization generation head is shown in Figure 2.

The polarization generation head is mounted on the positioning system platform with its rotation axis perpendicular to the pitch and yaw axis of the position system. In this configuration, the calibration system is capable of generating arbitrary polarizations with a user specified $\vec{k}$ which always intersects the neutral beam axis.

In addition to the calibration head, several other components have been developed that
FIG. 2. Polarization head assembly. The polarization generation head is capable of generating light with arbitrary polarization states using a variable liquid crystal retarder mounted at 45° relative to the transmission axis of a dichroic linear polarizer mounted on a motorized rotational stage. A red LED array behind an Opal glass diffuser generates a nearly Lambertian polarized plane source, or a laser diode generates a polarized ray coincident with the rotational axis of the system. All the components are mounted on a stepper-motor based precision rotational stage. An electronic inclinometer mounted to the stage base references the polarization parameters relative to gravity. 

...can be mounted on the stage platform, including: an integrating sphere to calibrate the spectral throughput of other beam based diagnostics; a bright unpolarized LED based Lambertian source to calibrate the throughput of the MSE system; a movable slit to calibrate the viewing footprint of beam-based diagnostics; and a laser system to perform precision metrology inside the C-Mod vessel. 

C. Calibration Work Flow

The robotic calibration system is controlled via TCP/IP and serial communications using a custom software package written in MATLAB. Signals are routed from a portable computer and electronics system in the C-Mod cell via cables to the calibration system inside the tokamak vacuum vessel. The computer controls the light source intensity and polarization parameters; positions the stages and the light source; triggers the MSE data acquisition...
FIG. 3. The MSE calibration system and its transportable support hardware outside the vessel. The system is controlled by a computer running a MATLAB program that communicates with the calibration system, triggers the MSE data acquisition and controls other processes.

system; sets the PEM retardances; sets the APD bias voltage, and has several analog and digital inputs and outputs for interfacing with other equipment. The control system contains logic which prevents movements that would cause collisions with other elements installed in the vessel or would damage the robotic components due to over-extension. The calibration system with control system is shown in Figure 3.

During calibration of the MSE diagnostic, the positioning system carries the polarization generation head to the intersection of an MSE sightline and the beam axis, and points the polarization generation head axis at the center of the MSE objective lens. This aligns the linear polarizer perpendicular to the mean sightline $\vec{k}$. The polarization head illumination area is large enough to fully fill the sightline viewing volume. The system then rapidly generates various polarizations while triggering the MSE data acquisition system. The system then typically moves to another MSE sightline but optionally it varies other parameters such as the light intensity, the PEM retardance or the temperature of optical components by activating heaters. The installed system, as it would appear during the calibration of an MSE sightline, is shown in Figure 4. The automated system has decreased the time required for a full calibration of all ten MSE sightlines from $>10$ hrs to $<3$ hrs and faster, slightly
FIG. 4. MSE calibration system installed invessel calibrating an MSE sightline. During the calibration, the polarization head is placed at the intersection of the beam and the MSE sightline to be calibrated and is pointed at the MSE objective lens, filling the sightline footprint with polarized light.

less accurate, modes of operation reduces this by a further factor of 8.

The Alcator C-Mod vacuum chamber is small and access is difficult, therefore the system is comprised of three pin-aligned components which fit through the narrow Alcator C-Mod vacuum entrance port where they are attached to sturdy, semi-permanent, ball-in-socket mounting points on the vacuum vessel. The system can be installed and removed from inside the vessel by a technician in approximately fifteen minutes without tools or a lifting system. Once installed, the calibration system performs an automated alignment procedure which is verified by pointing the laser at permanent landmarks inside the vessel. The MSE calibration typically requires a dark vessel and restricts manned access, precluding other maintenance activities. Therefore the system is often installed at the end the day shift, performs calibrations overnight without human oversight and is then removed in the morning.
to allow other work to continue. At the end of that day’s shift the system is re-installed to continue calibration with minimal loss of calibration accuracy. A digital video camera is incorporated into the positioning system with two additional cameras installed in the vessel to allow the system to be monitored remotely. The system performs a script of various calibration tasks, posts its progress in logs, and updates the MSE diagnostician via SMS text message. In addition to autonomous operation, the system can also be controlled remotely or from inside the vessel via a GUI on a small laptop or tablet with a wireless connection. The system has operated with minimal human interaction for over 72 continuous hours and has been used for MSE calibrations since 2009.

III. CALIBRATION OF ALCATOR C-MOD’S MSE DIAGNOSTIC

The robotic calibration system is used to obtain both the geometric calibration and the polarization calibration.

A. Geometry Calibration

The polarization angle from the motional Stark effect is a function of the projection of the sightline onto the electric field in the atom’s frame which is composed of the field from the plasma itself and the Lorentz field due to the neutral traversing the magnetic field at high velocity: \( \vec{E} = \vec{E}_{\text{plasma}} + \vec{v}_{\text{beam}} \times \vec{B}_{\text{plasma}} \). The linear polarization angle relative to the toroidal plane of the \( \sigma \) multiplet (\( \theta_{\text{pol}}^\sigma \)) emitted at a location in the plasma is thus a function of the local field components, beam velocity and viewing vector. The most general form for
the polarization angle is:

\[
\tan(\theta_{\text{pol}}^\sigma) = \frac{A_1 B_z + A_9 B_R + A_8 B_\phi + A_5 E_R/v + A_{10} E_\phi/v}{A_2 B_\phi + A_3 B_R + A_4 B_z + A_6 E_x/v + A_7 E_R/v + A_{11} E_\phi/v}
\]

\[A_1 = -\cos(\alpha + \Omega) \cos \beta\]
\[A_2 = \sin \alpha \cos \beta \cos \theta + \sin \beta \sin \Omega \sin \theta\]
\[A_3 = \cos \alpha \cos \beta \cos \theta - \sin \beta \cos \Omega \sin \theta\]
\[A_4 = \sin(\Omega + \alpha) \cos \beta \sin \theta\]
\[A_5 = -\cos \Omega\]
\[A_6 = -\cos \theta\]
\[A_7 = \sin \Omega \sin \theta\]
\[A_8 = -\sin \beta \cos \Omega\]
\[A_9 = -\sin \beta \sin \Omega\]
\[A_{10} = \sin \Omega\]
\[A_{11} = \cos \Omega \sin \theta\]

where \(\alpha\) is the angle between the beam velocity and local toroidal direction, \((\hat{\phi})\) in the horizontal plane; \(\beta\) is the angle between the beam velocity and the horizontal plane; \(\Omega\) is the angle between the sightline \(\vec{k}\) and the local toroidal direction in the horizontal plane; and \(\theta\) is the angle between the sightline and the horizontal plane and \(v\) is the beam scalar velocity. The geometry is shown in Figure 5 (a) and (b). Note that when the beam lies in the toroidal plane \((\beta = 0)\) \(A_1 - A_7\) are equivalent to those presented in Equation 2 in Rice\(^{24}\).

When the beam and sightline reside on the plasma midplane and the plasma electric field is neglected the result reduces to the standard simple dependence on the vertical and toroidal magnetic field:

\[
\tan(\theta_{\text{pol}}^\sigma) = \frac{-B_z \cos(\alpha + \Omega)}{B_\phi \sin \alpha}
\]

The polarization angle of the \(\pi\) emission and \(\sigma\) emission are exactly perpendicular when the Zeeman effect is neglected\(^{25}\).

The geometric calibration of a MSE sightline thus consists of determining the location of its viewing volume and the four angles \(\alpha, \beta, \Omega\) and \(\delta\). The calibration coefficients \(A_1 - A_{11}\) can then be derived and used in magnetic reconstruction programs such as EFIT\(^{26}\) to generate
the expected polarization angle at each viewing volume. This is then compared to the 
measured polarization angle from the diagnostic and the error is minimized to obtain a valid 
reconstruction.

When the 50 keV diagnostic neutral beam was installed inside the Alcator C-Mod ex-
perimental hall, a set of alignment targets and a laser beam were used to align the duct and beam tank. After the beam components were installed the beamline gate valve and calorimeter were opened, thereby allowing a laser beam to project through the entire beam system and onto the beam acceleration grids to confirm that the grids were centered on the duct axis. The beam alignment is monitored during beam operation in the duct using the beam calorimeter and at the beam strike point on the tokamak high-field side using a visible camera. During long duration maintenance periods the beam alignment has also been confirmed using IR imaging by firing the beam into castellated targets. During maintenance periods the alignment targets and laser are reinstalled in the beam duct and are used as a reference for the beam axis. The calibration positioning system is then installed and is aligned so that its translation path is collinear with, and the pitch and roll axes intersect with, the laser that traces the beam axis.

To determine the viewing volumes of the MSE sightlines, an illuminated 1 mm wide vertical slit is installed on the linear positioning stage in place of the polarization generation head and is moved in 1 mm steps along the beam trajectory while the data acquisition is triggered. The signal vs linear stage position accurately determines the locations along the beam axis where the MSE sightline crosses the beam trajectory as shown in Figure 5(c). The dual columns of fibers in each MSE sightline are evident as is the change in magnification from the optical axis (sightline 6) to the field extents (sightlines 1,10). The geometric calibration coefficients are then determined at the centroid of each sightline viewing footprint.

The positioning system’s capability to accurately measure relative angles using stepper motors with encoders is used to determine the geometric calibration angles. To measure the location of the stage inside the tokamak, the stage moves in the yaw axis in the horizontal plane until the laser of the polarization generation head is tangent to one side of the center column of the tokamak and then the stage rotates until the laser is tangent to the other side of the center column of the tokamak. The center column diameter is known very accurately, thus simple trigonometry yields the major radius at which the stage is located. The local toroidal direction is perpendicular to the bisector of the angle between the two tangent
The positioning system then points the laser through the targets in the beam duct and is then finally pointed at the center of the MSE objective lens. The calibration angles $\Omega$, $\delta$, $\alpha$ and $\beta$ are derived from the relative angles and the inclinometer output. The geometry is shown in Figure 5(a) and (b). Once the position and orientation of the positioning system is determined inside the vessel at a few positions via this technique, the system can reliably calculate the appropriate calibration angles and the major radius for any position along the beam trajectory. The accurate encoders in the system can also be used as a metrology tool to determine the absolute location of the diagnostic objective lenses or other features inside the vessel via triangulation from the opposite ends of the translation stage.

B. Polarization Calibration

The linear polarization calibration relates the measured signal intensities at the second-harmonic PEM frequencies to the polarization angle of the linearly polarized light incident on the diagnostic objective lens. In an ideal PEM-based polarimeter the polarization angle incident on the polarimeter is encoded in the amplitudes at the second harmonics of the PEMs ($I_{2\omega_1}$ and $I_{2\omega_2}$):

$$\frac{1}{2} \tan^{-1} \left( \frac{J_2(R_2) I_{2\omega_1}}{J_2(R_1) I_{2\omega_2}} \right) = \theta_{pol}$$

(3)

where $R_1$ and $R_2$ are the retardances of the first and second PEM respectively. Both retardances are usually set to be near 3.05 radians, which maximizes the signal intensities at the second harmonics of the PEMs and makes the system insensitive to small drifts in the PEM retardance. Typically we assume that the retardances of the two PEMs are equal and the error associated with this assumption is incorporated into the polarization calibration. Work is underway to incorporate on-line measurement of the PEM retardance into the data processing system.

Optical elements that are positioned in front of the polarimeter (lenses, mirrors, vacuum windows) modify the polarization state of the light incident on the polarimeter. In theory this can be accounted for using the Mueller matrix formulation for polarized light. Results have been discussed for a single non-ideal mirror$^{22,23}$. However, the inclusion of three non-ideal mirrors, many lenses and a non-ideal polarimeter (e.g. slightly mis-aligned PEM or linear polarizer, or unequal PEM retardances) presents a highly non-linear system with a
FIG. 5. The angles used in the calibration of an MSE sightline are calculated from relative angle measurements obtained by the robotic calibration system. The polarization generation head laser (red) is brought tangent to the two sides of the tokamak center column ($\psi_1$), then pointed though the beam alignment targets ($\psi_2$) and then pointed at the MSE objective lens ($\psi_3$) (b). The vertical angles of the view and beam $\theta$ and $\beta$ are measured using the inclinometer on the calibration system (c). The viewing volumes are measured by moving a 1 mm wide slit along the beam trajectory while triggering the MSE detectors (c). The calibration angles are determined at the centroids of the sightline viewing volumes.

large number of required parameters. Additionally, each diagnostic sightline is composed of many rays that strike optical elements at different locations and angles of incidence and thus have different polarization modifications. The calibration of the sightline is thus a weighted average of these rays. These effects cause Equation 3 to deviate from linear. It is unlikely the accuracy required for the calibration can be reliably achieved using a Mueller formalism with a-priori optical properties and geometry for a system this complex. Therefore an empirical technique is used here.

Polarization angles are modulo $\pi$ and therefore it is reasonable to assume the deviation from linear of the ideal polarimeter response can be represented with even harmonics similar
to a Fourier decomposition. In practice we find the polarization response of the Alcator C-Mod MSE diagnostic is extremely well fit using an offset, a linear term and the first two even harmonics:

\[
\frac{1}{2} \tan^{-1} \left( \frac{I_{2\omega_1}}{I_{2\omega_2}} \right) = B_0 + B_1 \theta_{\text{pol}} + B_2 \cos(2\theta_{\text{pol}} + 2\theta_{\text{pol}}) + B_4 \cos(4\theta_{\text{pol}} + 4\theta_{\text{pol}})
\]

with the coefficients \(B_x\) are found empirically by fitting the detected intensities at the PEM harmonics to the known input polarization angle \(\theta_{\text{pol}}\) from the polarization generation head.

Typically the polarization head is used to illuminate the MSE objective lens from a MSE sightline footprint with a set of \(\sim 35\) different polarization angles spanning \(\sim 340^\circ\) and the measured intensity ratios are fit to Equation 4, resulting in an rms residual fit error of \(< 0.03^\circ\). A sample data set is shown in Figure 6 where panel (a) shows the measured signal angle as a function of input polarization angle. Panel (b) shows the residual after including only \(B_0\) and \(B_1 = 1\) in the fit with the \(\cos(4\theta_{\text{pol}})\) structure evident. Panel (c) shows the residual after fitting with the offset and \(\cos(4\theta_{\text{pol}})\) terms with remaining \(\cos(2\theta_{\text{pol}})\) structure evident. Panel (d) shows the residual after fitting the full calibration function, (Equation 4). Note that there is no longer any periodic structure in the residual above the level of the individual angle measurement statistical uncertainty. Increasing the number of angles measured, increasing the range of angles measured, or changing the order of polarization angles has no effect on the computed fit coefficients or the magnitude of the residual error. The fit continues to yield small residuals even in extreme cases such as illuminating the MSE diagnostic with elliptically polarized light or operating MSE with non-optimized mirrors or non-equal PEM retardances.

Although the Fourier representation is used for empirical purposes, the terms can be shown to arise due to physical polarization effects using the Mueller formalism and numerical simulation of the polarimeter and optical system. \(B_0\) represents the angle of the polarimeter relative to gravity and the ideal (ie no phase shift, equal reflection ratios) angle rotation from reflection on the three mirrors in the MSE optical periscope. \(B_0\) also has a contribution from Faraday rotation, this contribution is small during operation of Alcator C-Mod due to the use of low Verdet constant glasses and is zero during invessel calibration. The measurement of this effect will be covered in a future publication. \(B_1\) accounts for imperfections in the polarization generation head and is typically found to be \(1 \pm 10^{-5}\) and is set identically
FIG. 6. The fitting of the polarimeter polarization response. The measured polarization angle (a) contains non-linear response due to non-ideal components in the polarization response (b). Subtracting off the dominate $\cos(4\theta_{pol})$ term leaves structure in the residuals (c). Including the first two even terms in the decomposition results in a small, structureless residual at the level of the uncertainty in the individual angle measurements (d).

to 1 for most work. $B_2$ arises from non-unity S and P reflection ratio in the mirrors and varies across the MSE view from nearly $0^\circ$ at the optical axis to $O(0.2^\circ)$ at the edges of the field. $B_4$ arises from multiple effects including non-equal PEM retardances, PEM-PEM fast-axis misalignment, PEM-linear polarizer misalignment, non-unity detector response at the two PEM frequencies, non-normal angle of incidence on the dual PEMS, non-zero phase shift after reflection from mirrors, and birefringence in the 10 MSE transmissive optics$^{28}$. This term is typically $O(0.5^\circ)$ on Alcator C-Mod and is largest near the the optical axis. The terms as a function of MSE sightline are shown in Figure 7. The terms vary campaign to campaign if the PEM assembly is removed and reinstalled.
FIG. 7. The sightline dependence of the fit coefficients. The offset term is monotonic (a) while the \( \cos(2\theta_{\text{pol}}) \) \( (B_2) \) and \( \cos(4\theta_{\text{pol}}) \) \( (B_4) \) dependent terms are parabolic, reaching a minimum and maximum respectively near the optical axis (OA). (b).

TABLE I. Variation of \( \text{mse} \) calibration coefficients when the calibration system was used to calibrate the same \( \text{mse} \) sightline on three different occasions. The calibration system was removed and re-installed between each trial. All values have units of degrees.

<table>
<thead>
<tr>
<th>Trial</th>
<th>( B_0 )</th>
<th>( B_4 )</th>
<th>( B_{4p} )</th>
<th>( B_2 )</th>
<th>( B_{2p} )</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>1\textsuperscript{st}</td>
<td>-62.640</td>
<td>0.254</td>
<td>228</td>
<td>0.058</td>
<td>235</td>
<td>0.021</td>
</tr>
<tr>
<td>2\textsuperscript{nd}</td>
<td>-62.623</td>
<td>0.327</td>
<td>223</td>
<td>0.058</td>
<td>238</td>
<td>0.013</td>
</tr>
<tr>
<td>3\textsuperscript{rd}</td>
<td>-62.626</td>
<td>0.313</td>
<td>224</td>
<td>0.057</td>
<td>238</td>
<td>0.014</td>
</tr>
</tbody>
</table>

|          | Stdev | 0.009 | 0.039 | 3 | 0.001 | 2 |
|          | Max-Min | 0.017 | 0.073 | 5 | 0.001 | 3 |

As shown in Table I, the calibration system is repeatable to 0.04° after removal and re-installation of the robot components and is repeatable to 0.02° in successive calibrations without component removal. The \( \text{mse} \) diagnostic polarization response drifts by < 0.05° over 48 hrs during tokamak maintenance periods when the tokamak is at atmospheric pressure and temperature. The automation, repeatability and rapid action of the robotic calibration system allows the dependence of the \( \text{mse} \) calibration on various parameters to be studied in detail by performing repeated calibrations as some parameter of interest is varied.

The use of fast digitization and numerical phased-locked loops in the \( \text{mse} \) analysis allows the intensity at all the PEM harmonics \((I_{1\omega_j}, I_{2\omega_j} \ldots I_{n\omega_j})\), including sets of mixed harmonics
(i.e. \( I_{3\omega_2-1\omega_1} \)), to be precisely determined. This is routinely done on Alcator C-Mod for each plasma discharge and calibration shot. Decomposing the output of the polarimeter as a function of input polarization angle into even Fourier components is also shown to be valid for the intensities at the various PEM harmonics. Each of the \( n \) PEM harmonics are fit the equation:

\[
I_{n\omega_1,2} = C_0 + C_2 \cos(2\theta_{\text{pol}} + C_{2p}) + C_4 \cos(4\theta_{\text{pol}} + C_{4p})
\]

(5)

where \( C_2 \) is typically \( \sim 100 \) times larger than \( C_0 \) or \( C_4 \). A sample of this process is shown in Figure 8. Panel (a) shows the absolute value of the measured intensity at various PEM harmonics which have been normalized by \( \sqrt{I_{2\omega_1}^2 + I_{2\omega_2}^2} \) averaged over all the input angles. The primary dependence is \( \cos(2\theta_{\text{pol}}) \) as seen by the small residuals in panel (b) after accounting for this term. Note that the 1st (red, asterisks), 3rd (orange, triangle) and 4th (green, square) harmonics are at or below the digitizer bit resolution (dark grey dashed line) while the 2nd harmonic (blue, diamond) remains above this level. Including all the terms in the Equation 5 results in a small, structureless residual (c) for all harmonics. Thus the entire system response can be accurately distilled into a table of coefficients, five per PEM harmonic.

C. Spectral Calibration

The spectral passbands of the optical filters for the MSE diagnostic on Alcator C-Mod were chosen to view the \( \pi \) multiplet of full-energy beam component, because the full energy \( \sigma \) multiplet is contaminated by emission from half-energy and third-energy \( \pi \) components. Therefore, the optimal spectral location of the narrow bandpass filters depends on both the beam energy and the magnetic field strength at the location of the MSE viewing volume via the Doppler shift and Stark split. Proper positioning of the filter bandpass over the \( \pi \) emission is thus important, and the optimum passband wavelength changes when the strength of the C-Mod toroidal magnetic field is changed. The filters are held in thermal ovens which collimate the light from a MSE sightline fiber bundle. The ovens can be temperature tuned from room temperature to 60 °C which shifts the center wavelength to the red due to thermal expansion. Due to the large etendue of each C-Mod MSE sightline, the ovens are designed so
FIG. 8. The fitting of the intensities at the first four PEM harmonics. The measured intensities at different PEM harmonics (a). All the intensities have been normalized by $\sqrt{I_{2\omega_1}^2 + I_{2\omega_2}^2}$ averaged over all the input angles. The residual after fitting the offset and term varying by $\cos(2\theta_{pol})$ (b). The residual after fitting all the terms in the Equation 5 (c). The digitizer bit resolution is shown as the grey dashed line.

that the 50 mm diameter filter is nearly fully illuminated. Additionally, due to the large number of fibers in a sightline, the light from the fiber bundle is incident on the filter with a cone angle of $5^\circ$. This range of incident angles slightly widens and shifts the effective filter bandpass to the blue and decreases the maximum transmission relative to the manufacturer measurements, which were performed only on the center of the filter with highly collimated
light. To take account of these effects and to determine the temperature tuning coefficients, the filter responses were measured in-situ.

The filter in the oven was illuminated using variable-sized field and aperture stops. The light passed through the filter and was collected by an integrating sphere which was connected by a fiber to a high resolution spectrometer. As expected, the filter bandpass has a slight dependence on the illumination configuration as shown in Figure 9. When illuminated with the same illumination pattern as the MSE system, the filter bandpass centers varied by as much as 0.2 nm from that reported by the manufacturer.

To determine the effect of filter heating, the oven was commanded to an elevated temperature and the filter bandpass was measured over a period of hours while the heat diffused into the filter until it reached a new equilibrium temperature. In steady-state, the filter passband shifted to the red when heated with a coefficient of $0.018 \pm 0.001 \text{ nm/}^\circ\text{C}$. The dynamic heating test determined that a delay of 4 minutes is required for the filter to equilibrate to the measured oven temperature, providing confidence that MSE can change filter settings between Alcator C-Mod plasma shots ($\sim 15$ min). The results are shown in Figure 10.

The measured filter responses are integrated into software which controls the filter ovens to account for the Stark split and Doppler Shift, using the geometric calibration, the local toroidal magnetic field and the applied beam acceleration voltage. The beam voltage is confirmed using Doppler spectroscopy inside the neutral beam tank. The entire spectral system has been verified by performing beam into gas experiments with no field (and thus no Stark split) in which the filter temperature is changed on successive shots to move the passband across the Doppler shifted beam emission.

IV. CONCLUSIONS

A robotic calibration system has been developed for Alcator C-Mod’s MSE diagnostic which is able to illuminate the diagnostic with polarized light from known positions along the path of the Alcator C-Mod diagnostic neutral beam during maintenance periods. The system consists of a positioning system capable of precise 3D alignment inside the vessel and a polarization generation head which creates arbitrary polarization states with orientations known to $< 0.05^\circ$. The polarization generation head can illuminate the diagnostic with a Lambertian source, fully filling the diagnostic viewing volume or a laser source to simulate
FIG. 9. Measurement of filter spectral response. The filters were illuminated with variable etendue using a field and aperture stop on the collimating system. The bandpass was measured using a high resolution spectrometer after collecting the light with an integrating sphere. As the etendue is increased, the filter center (*) and filter width (○) experience a systematic shift to the blue and a widening of the passband respectively.

Using triangulation, the robotic calibration system can also self-consistently determine the geometric calibration necessary for interpreting the polarization measurement. Due to the complex optical periscope in Alcator C-Mod’s MSE diagnostic the polarization angle calibration is captured using an empirical fit instead of an a-priori Mueller matrix representation. This quasi-Fourier decomposition captures the calibration of the diagnostic yielding small residuals ($< 0.03^\circ$) using only the first two even terms in the input polarization angle and continues to work well when the diagnostic is operated in unusual configurations. The same methodology is extended to the individual PEM harmonics with small residuals. Combined with precise spectral measurements of the narrow bandpass filters and their tuning coefficients, all of the components necessary for the MSE calibration are determined. The
FIG. 10. Measurement of the temperature response of the filters. The MSE filters were heated dynamically in their ovens while their bandpass was measured with a spectrometer. The filter temperature was inferred from the measurement of the bandpass using previously determined temperature tuning coefficients (solid). The filter temperature lags the oven temperature (dotted) by 4 minutes (dashed).

The robotic system has been used to reliably perform over 50,000 calibration data cycles to date. The automation, speed and repeatability of the robotic calibration system has allowed systematic errors in the MSE polarization measurement to be studied extensively and work is ongoing to evaluate the systematics and sensitivities in the diagnostic as well as to optimize the diagnostic operation. Unlike other MSE calibration techniques, these techniques do not require any plasma or beam-into gas discharges, eliminating impacts to machine operation due for diagnostic calibration.

Although the robotic calibration technique does not require machine operation time, the diagnostic response and geometry must be stationary during the plasma experiments for the calibration to be properly applied. Comparisons with previously developed plasma-based calibration techniques\textsuperscript{21} show that the diagnostic response varies due to the heating and cooling of the optical periscope causing stress-induced birefringence in the transmissive
MSE optical components. Efforts to stabilize the thermal environment and complementary calibration techniques have been developed and fielded and are awaiting confirmation.

In future and current fusion devices it will be increasingly important to determine precise diagnostic calibrations without using machine operation time, to determine the diagnostic’s susceptibility to systematic errors, and to develop mitigation strategies. Therefore the development of calibration systems that can be used at various stages in the diagnostic deployment such as during development, in test environments, and in-situ is important to ensure reliable diagnostic results.

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Appendix A: Calibration of the Transmission Axis of the Linear Polarizer

The polarization angle of the light produced by the polarization generation head has multiple contributions:

\[
\theta_{\text{polarization}} = \theta_{\text{level angle}} + \theta_{\text{stage angle}} - \theta_{\text{TA offset}}
\]  

(A1)

where \( \theta_{\text{level angle}} \) is the angle of the stage base with respect to gravity measured by the electronic inclinometer, \( \theta_{\text{stage angle}} \) is the angle of the polarization generation stage relative to its base which is known to high accuracy from the stepper-motor encoder and \( \theta_{\text{TA offset}} \) is the angle of the transmission axis of the linear polarizer relative to the rotating part of stage on which it is fixed. The later angle is not known a-priori, thus a bench calibration using Malus’s law has been performed to measure it. A laser shines through a beam expander then through a high quality polarizing beam splitter which is placed on a stage and leveled relative to gravity using a high precision digital level. The laser beam then shines through the linear polarizer mounted on the polarization generation head (with the sources removed) and is then condensed and detected by a photodiode with a trans-impedance amplifier. The
FIG. 11. \( \theta_{TA \text{ offset}} \) is measured using a Malus’s law setup in which a laser beam is expanded to \( \sim 1 \) cm diameter and passed through a polarizing beam splitter on a stage leveled to gravity before being passed through the polarization generation head and then condensed onto a photodiode. The polarization generation head can rotate the linear polarizer installed on it. The angle between the beamsplitter and linear polarizer faces can be varied by rotating the beamsplitter slightly.

The polarization head is commanded through a series of \( \sim 600 \) angles spanning several revolutions and the voltage from the photodiode, \( \theta_{\text{level angle}} \) and \( \theta_{\text{stage angle}} \) are recorded. The resulting voltage follows Malus’s law:

\[
\frac{I_{\text{transmitted}}}{I_{\text{incident}}} = ER + (1 - ER) \cos^2(\theta_{\text{beam splitter}} - \theta_{\text{polarization}}) \tag{A2}
\]

where ER is the extinction ratio of the polarizer pair and \( \theta_{\text{beam splitter}} = 0^\circ \) or \( 90^\circ \) depending on the orientation of the beam splitter. The voltage from the photodiode is then fit to the equation:

\[
I_{\text{transmitted}} = I_{\text{incident}}(ER + (1 - ER) \times \\
\cos^2(\theta_{\text{beam splitter}} + \theta_{\text{level angle}} + \theta_{\text{stage angle}} - \theta_{\text{TA offset}})) \tag{A3}
\]

with \( I_{\text{incident}}, ER \) and \( \theta_{\text{TA offset}} \) as fitting parameters. A sample set of data and the resulting fit residuals are shown in Figure 12. The fit is performed over various angle ranges from \( \pm 30^\circ \) to \( \pm 360^\circ \) and we observe that the fit parameters are independent of which subset of data the fit is used. The fit parameters are also independent of which direction the stage
FIG. 12. The light intensity as the polarizer is rotated is fit using Equation A3 to determine the \( \theta_{TA \ offset} \) (a) yielding small residuals (b).

Finally, the fit parameters are invariant if the angles are randomly chosen, indicating there is no significant hysteresis in the polarization head operation. The stability of the laser source and detector was monitored and found to vary < 0.1% over the timescale of the data acquisition (~ 0.5 hr). The test was done at multiple detector orientations with negligible difference, indicating that the detector is polarization insensitive.

The measured \( \theta_{TA \ offset} \) depends on the angle between the beamsplitter face and the linear polarizer face due to a projection. Both of the faces are plumb, leaving one free parameter. The stage on which the beam splitter sits was rotated slightly about its vertical axis to scan the angle between these two faces. The beam splitter is then flipped 180° about the vertical axis and the AOI scan is repeated reversing the projection. The cube is then flipped 180° about the laser axis and the process is repeated. This yields four sets of measured \( \theta_{TA \ offset} \) vs AOI. The two faces are most parallel where the curves cross, yielding an accurate estimate of \( \theta_{TA \ offset} \). This process is shown in Figure 13. The process was repeated with a beam splitter from another manufacturer yielding consistent results.

The spatial variation of \( \theta_{TA \ offset} \) across the face of the polarization generation head was determined by varying the location at which the expanded laser beam impacts the linear polarizer. The laser beam traces out an annulus on the face of the linear polarizer as the polarization generation head rotates, thus the fit is then performed over 90° portions of the
FIG. 13. The angle between the beamsplitter face and the linear polarizer face is varied by rotating the beam splitter about its vertical axis, this results in a change in the measured $\theta_{\text{TA offset}}$ (a-b). The beam splitter is flipped 180° about the vertical axis and the scan is repeated (c-d). The beam splitter is then rotated 180° about the laser trajectory and the process is repeated again (c-b, a-d). Where the four sets of data cross the two faces are most parallel yielding an accurate $\theta_{\text{TA offset}}$.

This results in a measured variability of $\theta_{\text{TA offset}}$ (within the central 15 mm radius used to calibrate MSE) of $\pm0.05^\circ$ which is consistent with previous measurements of high performance dichroic linear polarizers\textsuperscript{30}. In the future the dichroic linear polarizer will be replaced by a wire grid polarizer to reduce this variability, which is the largest source of uncertainty in this technique.

REFERENCES


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