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Millimeter Wave Scattering and Diffraction in 110 GHz Air Breakdown Plasma

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Abstract: We present measurements of the scattering, reflection, absorption, and transmission of a 1.5 MW, 110 GHz quasioptical gyrotron beam by a self-induced air breakdown plasma. The breakdown forms a periodic array of plasma filaments propagating toward the gyrotron source and oriented parallel to the incident electric field polarization. For an incident intensity of 3 MW/cm², calorimetric measurements show that about 45% of the microwave beam power is absorbed by the plasma, 1% is reflected directly backward, and the remainder is diffracted into a wide angular distribution. The far-field diffraction patterns, which show distinctive sidelobes, have been measured as a function of time during the discharge. The measured patterns show good agreement with a simple model of absorption by the plasma as the plasma varies in size during the breakdown pulse. We also observe that approximately 10 times more power is scattered in the direction perpendicular to the filaments than parallel.

Key words: gyrotron, millimeter wave, air breakdown, quasioptical

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I. Introduction

Megawatt-level millimeter wave beams, produced by gyrotrons, have applications in key areas of plasma science. For example, tokamak devices rely on absorption of millimeter waves via electron cyclotron resonance to heat plasma for fusion [1, 2]. In that case, a quasioptical beam is formed to couple the radiation into a low-loss overmoded transmission line [3, 4]. Other applications, such as plasma propulsion [5] or stand-off detection, [6] use a beam focused into atmospheric air at high intensity to create an electric discharge plasma.

When a quasioptical beam is intense enough to induce an electrical discharge in a gas, the resulting plasma distorts the beam [7, 8]. Reflection of the incident beam from the plasma can cause the ionization front to move back toward the microwave source, as the combined intensity of the incident and reflected waves initiates a new discharge [9]. Because the incident beam is near the intensity threshold for breakdown, this small amount of reflected power raises the field in front of the filament above threshold and is responsible for the unique formation of filaments at λ/4 wavelengths [10, 11]. Scattering and diffraction of the beam occur when the beam is strongly focused, such that the discharge plasma is smaller than or comparable in size to the focused beam.

Recent experiments have studied air breakdown plasmas produced by a focused Gaussian beam of 110 GHz millimeter waves. [12, 13] The megawatt-level beam produces a periodic array of plasma filaments that propagates in discrete steps toward the microwave source, due to reflection of the incident beam by the plasma. In this paper, we present measurements of the scattering, reflection, transmission, and absorption of the beam by the breakdown plasma. We observe that significant fractions of the incident
beam power are transmitted and absorbed, and most of the remaining power is scattered at large angles to the beam direction.

The paper is organized as follows. Section II describes the experimental setup and the raw data collected. Section III presents a simple plasma absorption model to describe the far-field diffraction pattern and the power balance observed at different times in the plasma lifetime. Section IV discusses the conclusions of this study.

II. Experiment

The experimental setup is illustrated in Fig. 1(a). A linearly-polarized Gaussian millimeter wave beam is generated by a 1.5 MW, 110 GHz gyrotron oscillator, operated at peak power levels in the range 800-900 kW [14]. The beam is focused to a small spot (1/e spot radius $w_0 \sim 4$ mm) by a high-density polyethylene lens, reaching a high enough intensity to cause breakdown at atmospheric pressure (peak intensity $\sim 3$ MW/cm$^2$). A radiofrequency (rf) diode detector mounted on a rotating arm is pointed at the breakdown and moved through a large range of angles, observing microwave power that is transmitted and scattered by the plasma. The diode is fed by an open-ended WR08 rectangular waveguide. A variable attenuator is used in conjunction with the diode to provide -30 dB of dynamic range relative to the peak incident beam intensity. A calorimeter is used for absolute-scale measurements of the average transmitted and scattered power, allowing the absorbed power to be estimated. From the diode measurements it is determined that approximately 1% of the incident peak power is reflected into a backward cone of ±7º.
Figure 1. (color online) (a) Experimental setup. (b) Time-integrated grayscale photo of plasma in $yz$ plane. (c) Time-integrated photo of plasma in $xz$ plane.

Fig. 1(b) is a time-integrated photograph of the breakdown in the $yz$ plane, showing the tops of the plasma filaments oriented along the electric field polarization. Fig 1(c) is a photograph in the $xz$ plane, showing a side view of the filaments. Angular intensity distributions of scattered and transmitted power are measured in $yz$ (perpendicular to filaments) and $xz$ (parallel to filaments). The opening of the rectangular waveguide connected to the diode detector is oriented with the broad wall dimension always perpendicular to the E field polarization. To enable the $xz$-plane measurements, an extra mirror is added to the setup to rotate the incident polarization by 90 degrees.
Fig. 2(a) shows examples of the power waveforms measured by the diode before the beam focal plane (incident power) and downstream of the breakdown plasma (transmitted power) on the beam axis. The start of the breakdown is measured as a distinct perturbation in the diode waveform, e.g. estimated at 2.5 μs for the waveform in Fig. 2(a) [15]. Fig. 2(b) shows typical diode waveforms measured at various angles in the yz plane. The temporal structure of the peaks and dips in the waveform depends on the observation angle. At smaller angles, ≤ 10 deg., the transmitted power decreases monotonically in time similar to the on-axis case. At larger angles, ≈ 10-45 deg., a high or low power level is measured depending on the time during the pulse, and is very repeatable at a fixed angle. At angles > 45 deg., the peak signals are weaker by ~20 dB and the temporal structure varied significantly shot-to-shot.

The repeatability of the waveforms at angles < 45 deg. allows us to compile the power levels measured at a given time value into an angular intensity distribution. We thus measure the far-field diffraction pattern of the beam transmitted through the plasma by recording the diode signal levels measured between ±40 deg. at a fixed time after the start of the breakdown. Fig. 3 compares the measured angular distributions in the yz and xz planes, 1 μs into the breakdown. The angular distribution measured without the plasma is shown for reference. The symmetric side lobes of a diffraction pattern are clearly seen at approximately ±20 degrees in the yz plane measurement with plasma present (Fig. 3(a)). Near zero degrees, the transmitted intensity is reduced by 0.5-1.5 dB relative to the incident beam. Figure 3(b) shows the same measurement in the xz plane; the side lobe structure is much less distinct than in the yz plane.
Figure 2. (color online) (a) Measured incident power (dashed black line) waveform compared with transmitted power measured at $\theta_y = 0$ deg (solid blue line). Breakdown starts at 2.5 $\mu$s. (b) Power waveforms measured by diode placed at $\theta_y = 10 – 170$ deg.
Figure 3. (color online) Angular intensity distribution (black circles) measured in (a) $yz$ plane and (b) $xz$ plane, relative to peak incident beam intensity, 1 μs after breakdown. Solid blue line shows Gaussian beam intensity distribution measured with no breakdown plasma.
The scattered power is measured by scanning the diode at large angles, -40 to -170 degrees. The detected waveform exhibits multiple peaks in time, as shown in Fig. 2b. Because the temporal structure of the scattered power varies shot-to-shot, the shot-to-shot average of the peak signal value during the entire breakdown event was recorded at each angle. Fig. 4 shows the result of this measurement in the yz and xz planes. The detected power level in the yz plane is ~5-10 dB above that in the xz plane. The peak scattered power in the yz plane occurs near -100 degrees, and is -15 dB below peak beam intensity. In the xz plane, a minimum scattered power level of -30 dB occurs near -70 degrees. The reflected power level, measured nearly straight backward at -170 degrees, is approximately -22 dB below peak intensity. From the measurement of the reflected power, we estimate that a total of about 1% of the incident power is reflected directly back towards the source.

A calorimeter is placed in different positions next to the breakdown plasma, to measure the absolute power levels averaged over the entire pulse. The full beam average power incident to the breakdown region, measured before the focal plane, is $2.6 \pm 0.1$ W for a 3 μs gyrotron pulse (0.87 MW peak power) at a repetition rate of one pulse per second. Placing the calorimeter directly downstream of the breakdown, subtending an angle of 90 degrees, we measure an average transmitted power of $1.2 \pm 0.1$ W, which is 46% of the incident power. Placed at 90 degrees to the beam direction and facing the plasma (Fig. 1a), 0.1 W is observed scattered in the yz plane and 0.01 W scattered in the xz plane; this 10 dB difference is consistent with the diode measurements of instantaneous scattered power in Fig. 4. The calorimeter measurements provide order-of-magnitude confirmation of the ~20 dB relative intensity difference between the peak
transmitted beam (Fig. 3) and the scattered power (Fig. 4). From all of these measurements, we then estimate that about 46% of the power is transmitted into the forward direction; about 9% of the power is scattered in the transverse direction and about 1% is reflected directly back towards the source. The missing power, amounting to about 45% of the incident power, is absorbed.

Figure 4. (color online) Peak scattered power measured in $yz$ (red circles) and $xz$ (black diamonds), relative to peak incident beam intensity, as a function of angle.

III. Absorption-Diffraction Model

We use a simplified plasma model to calculate the main features of the instantaneous far-field diffraction pattern of the gyrotron beam passing through the breakdown plasma. The plasma region is modeled as a cylinder oriented along the $z$ axis, with length $L_p$ and radius $r_p$ determined experimentally by fast-gated photography, as shown in Fig. 5. Within the cylinder, the plasma density distribution is modeled as uniform in $z$ and Gaussian in $r$ as given by the expression $n(r, z) = n_0 \exp(-r^2 / r_p^2)$. $n_0$ is taken to be
0.1 \( n_c \), where \( n_c = \left( \omega^2 + v^2 \right) e_0 m_e / e^2 = 4 \times 10^{21} \text{ m}^3 \) is the critical density including collisions at \( f = 110 \text{ GHz} \), where \( v = 5.3 \times 10^9 \text{ p [Torr]} \) is the collision frequency in Hz at a pressure \( p \). This value of \( n_0 \) is consistent with values derived in more detailed theories [16, 17] and will be found to give reasonable agreement for our simplified model. This simplification effectively replaces the density distribution of a closely-spaced group of plasma filament structures with an assumed average density distribution, in order to estimate the total wave absorption near the beam center. Because the plasma radius is less than the beam radius, attenuation of the microwave beam by absorption distorts the beam field distribution in a non-uniform way. The distorted field distribution is calculated at the downstream end of the plasma by considering the plasma complex permittivity, and a Fraunhofer integral is used to find the far-field angular intensity pattern at the measurement position. For simplicity, we assume for this calculation that the plasma is cylindrically symmetric. We compare the results of this 1D calculation with measurements of the diffraction pattern in the \( yz \)-plane, which was more readily measured than that in the \( xz \)-plane (Section II).
Figure 5. (a) Diagram of plasma absorption model, showing Gaussian beam envelope (dashed line) and plasma cylinder. The beam propagates in the $z$ direction. The small axes show the radial plasma density distribution inside the cylinder. (b) Fast-gated images taken in the $yz$ plane using 10 ns shutter, at 150 ns, 300 ns, 1 $\mu$s, and 2 $\mu$s after the beginning of breakdown. The measured values of $r_p$ and $L_p$ are shown to the right.

The amplitude of the incident microwave beam before passing through the approximately cylindrical plasma is given by

$$A_0(r,z) = E_0 \exp(-r^2/w^2)$$

(1)

where

$$w^2 = w_0^2 (1 + z^2/z_0^2)$$

(2)

$$z_0 = \pi w_0^2 / \lambda$$

(3)
Since the waist size $w_0 = 4$ mm, $z_0 = 18$ mm and $z_0 > L_p$, and we may use $w = w_0$ in Eq (1). Assuming a cold, collisional plasma, the dielectric constant in the plasma is

$$\varepsilon = 1 - \frac{n}{n_c} \left(1 + i \frac{V}{\omega} \right)$$  \hspace{1cm} (4)

where

$$n(r) = \begin{cases} n_0 \exp \left(- r^2 / r_p^2 \right) & r < r_p \\ 0 & r > r_p \end{cases}$$  \hspace{1cm} (5)

is the cylindrical plasma density profile. Using the above expression, the amplitude of the microwave beam after passing through the plasma is given by the expression

$$B(r, L_p) = \begin{cases} A_0(r) \exp \left(- ikL_p \sqrt{1 - \frac{n}{n_c} \left(1 + i \frac{V}{\omega} \right)} \right) & r < r_p \\ A_0(r) \exp \left(- ikL_p \right) & r > r_p \end{cases}$$  \hspace{1cm} (6)

where $k = 2\pi / \lambda$ is the free-space wavenumber of the millimeter waves. Figure 6 shows the radial microwave amplitude $|B(r, L_p)|$ for the various values of $r_p$ and $L_p$ which corresponds to the physical dimensions of the plasma at 150 ns, 300 ns, 1 $\mu$s, and 2 $\mu$s.
Figure 6. Radial microwave amplitude $|B(r, L_p)|$ for a) $r_p=1.4 \text{ mm}$ and $L_p=10 \text{ mm}$, b) $r_p=1.9 \text{ mm}$ and $L_p=5 \text{ mm}$, c) $r_p=4.3 \text{ mm}$ and $L_p=4 \text{ mm}$, and d) $r_p=5.7 \text{ mm}$ and $L_p=3 \text{ mm}$ corresponding to the physical dimensions of the plasma at 150 ns, 300 ns, 1 μs, and 2 μs, respectively.

In the case where both $n/n_c << 1$ and $(n v_e / n_c \omega)^2 << 1$, Eq. (4) takes on the simplified form
\[ B(r, L_p) = \begin{cases} 
A_0(r) \exp(-ikL_p) \exp \left( -\frac{1}{2} kL_p \frac{nv}{n_c} \right) & r < r_p \\
A_0(r) \exp(-ikL_p) & r > r_p 
\end{cases} \]

which is in general true for all of the values of \( n, r_p, \) and \( L_p \) considered in this paper.

The far-field diffraction pattern of the beam exiting the plasma is calculated by the Fourier transform of the radial microwave amplitude \( B(r, L_p) \),

\[ F(r, z) = \frac{1}{z} \int_0^\infty B(r', L_p) J_0(k r' / z) k r' dr' \quad (7) \]

At a distance \( z \gg r \) from the plasma, the angle is given by \( \theta = r / z \). The final form of the angular intensity pattern at distance \( z \), relative to the peak beam intensity at \( r = 0 \), is given by

\[ F(\theta) = \left| \frac{\int_0^\infty B(r') J_0(k \theta r') r' dr'}{\int_0^\infty B(r') r' dr'} \right| \quad (8) \]

Figure 7 shows the measured and calculated angular intensity patterns (\( \sim F^2(\theta) \)) of the beam transmitted through the plasma, at different times during the breakdown. At each time, the values of the plasma dimensions \( r_p \) and \( L_p \) were taken from Fig. 5; at earlier times (150 ns and 300 ns) the effective radius of the plasma is small, and the far field diffraction pattern exhibits a wide central peak and a single side lobe in the range of the measurement. At later times, when the size of the plasma is large, the central peak narrows and more side lobes are observed. The data show good overall agreement with the calculated diffraction patterns. There is excellent quantitative agreement in the shape and relative amplitude of the central peak of the diffraction patterns and the positions of
the first minima at each time. There is good agreement in the positions of the subsequent
minima.

**Figure 7.** (color online) Measured far-field diffraction patterns (circles) of beam passing through plasma at (a) 150 ns, (b) 300 ns, (c) 1 µs, and (d) 2 µs after the beginning of breakdown. The calculated unperturbed diffracted Gaussian beam (dashed blue line) is shown for reference against the calculated diffraction pattern from the absorption-diffraction model (solid red line).
IV. Discussion and Conclusion

We present the results of the measured scattered, reflected, absorbed and transmitted power in air breakdown induced by an intense 110 GHz beam from a gyrotron. Measurements of the scattered power show good agreement with a simplified model of a cylindrical collisional plasma with uniform plasma density along the axis and a Gaussian profile transverse to the axis.

While the far-field diffraction measurements do not resolve the sub-wavelength periodic structure of the breakdown plasma, the filamentary geometry is manifested in the measured scattered power. The scattered power observed in the $yz$ plane was approximately 10 times that observed in the $xz$ plane, indicating that the linearly polarized millimeter waves preferentially scatter perpendicular to the filament orientation and polarization direction $\hat{\mathbf{x}}$. This may be due to the sub-wavelength plasma filaments acting as dipole scattering sources, which has been studied previously as a microwave diagnostic technique [18].

The percentage of power observed reflected backward is small. This is likely because the breakdown plasma subtends a limited portion of the beam, $< 50\%$ by area. The amount of power reflected (at $\sim 170$ deg) is seen to grow significantly at later times in the breakdown ($1 – 2 \mu$s, Fig. 2(b)), when the transverse size of the plasma is larger. In addition, some of the reflected power may be reabsorbed in filaments and filaments in the process of forming. Only a small percentage reflection is needed to raise the electric field above the breakdown threshold, since the ionization frequency $\nu_i$ in air depends on the incident field $E$ as $\nu_i \propto E^{5.33}$ [19].
These findings have important implications for applications in which millimeter wave radiation scattered from air breakdown plasma is measured as a means of stand-off detection or characterization [20]. The beam polarization and plasma size may need to be considered when choosing detector placement and designing the rf diagnostics.
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References


